Exercise 13.1

Q1

Three angles of a quadrilateral area respectively equal to $110°, 50°$ and $40°$. Find its fourth angle.

Solution

Let fourth angle be x.

We have,

Sum of all angles of a quadrilateral = $360°$

$\Rightarrow 110° + 50° + 40° + x = 360°$

$\Rightarrow 200° + x = 360°$

$\Rightarrow x = 160°$

Q2

In a quadrilateral $ABCD$, the angles A, B, C and D are in the ratio $1:2:4:5$. Find the measure of each angle of the quadrilateral.

Solution

Let the angles of the quadrilateral be $A = x, B = 2x, C = 4x$ and $D = 5x$. Then,

$A + B + C + D = 360°$

$\Rightarrow x + 2x + 4x + 5x = 360$

$\Rightarrow 12x = 360$

$\Rightarrow x = 30$

$\therefore A = 30°, B = 60°, C = 120°$ and $D = 150°$

Q3

The angles of quadrilateral are in the ratio 3: 5: 9: 13, Find all the angles of the quadrilateral.

Solution

Let the common ratio between the angles is x. So, the angles will be 3x, 5x, 9x and 13x respectively.

Since the sum of all interior angles of a quadrilateral is 360^o.

$\therefore\ 3x + 5x + 9x + 13x = 360^o$

$30x = 360^o$

$x = 12^o$

Hence, the angles are

$3x = 3 \times 12 = 36^o$

$5x = 5 \times 12 = 60^o$

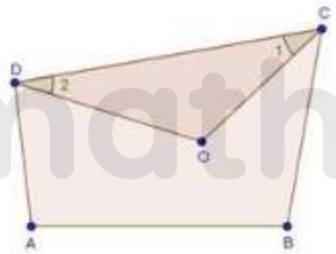
$9x = 9 \times 12 = 108^o$

$13x = 13 \times 12 = 156^o$

Q4

In a quadrilateral $ABCD$, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$.

Solution



In ΔDOC

$\angle 1 + \angle COD + \angle 2 = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow \angle COD = 180 - \angle 1 - \angle 2$

$\Rightarrow \angle COD = 180 - (\angle 1 + \angle 2)$

$\Rightarrow \angle COD = 180 - \left(\frac{1}{2}\angle C + \frac{1}{2}\angle D\right)$ [$\because OC$ & OD are bisectors of $\angle C$ & $\angle D$ respectively]

$\Rightarrow \angle COD = 180 - \frac{1}{2}(\angle C + \angle D)$ —(i)

In quadrilateral $ABCD$

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Angle sum property of a quadrilateral]

$\angle C + \angle D = 360 - (\angle A + \angle B)$ —(ii)

Substituting (ii) in (i)

$\Rightarrow \angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$

$\Rightarrow \angle COD = 180 - 180 + \frac{1}{2}(\angle A + \angle B)$

$\Rightarrow \angle COD = \frac{1}{2}(\angle A + \angle B)$

Exercise 13.2

Q1

Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(50 - x)^\circ$. Find the measure of each of the parallelogram.

Solution

Since opposite angles of a parallelogram are equal

$\therefore 3x - 2 = 50 - x$

$\Rightarrow 3x + x = 50 + 2$

$\Rightarrow 4x = 52$

$\Rightarrow x = 13$

$\therefore (3x - 2)^\circ = (3 \times 13 - 2)^\circ = 37^\circ$

$(50 - x)^\circ = (50 - 13)^\circ = 37^\circ$

Adjacent angles of a parallelogram are supplementary.

$\therefore x + 37^\circ = 180^\circ$

$\therefore x = 180^\circ - 37^\circ = 143^\circ$

Hence, four angles are : $37^\circ, 143^\circ, 37^\circ, 143^\circ$

Q2

If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution

Let the measure of the angle be x

$\therefore$ The measure of the adjacent angle is $\frac{2x}{3}$

Since the adjacent angle of a parallelogram is supplementary

Hence, $x + \frac{2x}{3} = 180^0$

$\Rightarrow \frac{5x}{3} = 180^0$

$\Rightarrow x = \frac{180^0 \times 3}{5} = 108^0$

Adjacent angles are supplementary

$\therefore 108^\circ + x = 180^\circ$

$\Rightarrow x = 72^\circ$

Hence, four angles are : $108^\circ, 72^\circ, 108^\circ, 72^\circ$

Q3

Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

Solution

Let the smallest angle be x

$\therefore$ the other angle is (2x − 24)

Now,

$x + 2x - 24 = 180$ [$\because$ Sum of adjacent angle of a parallelogram is 180°]

$\Rightarrow 3x - 24 = 180$

$\Rightarrow 3x = 180 + 24$

$\Rightarrow 3x = 204$

$\Rightarrow x = \frac{204}{3} = 68^{\circ}$

$x = 68^{\circ}$

$\Rightarrow 2x - 24^{\circ} = 2 \times 68^{\circ} - 24^{\circ} = 136^{\circ} - 24^{\circ} = 112^{\circ}$

Hence, four angles are : 68°, 112°, 68°, 112°

Q4

The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side?

Solution

Let the shorter side be x

$\text{Perimeter} = x + 6.5 + x + 6.5$ [Sum of all sides]

$22 = 2(x + 6.5)$

$11 = x + 6.5$

$x = 11 - 6.5 = 4.5$ cm

$\therefore$ Shorter side = 4.5 cm.

Q5

In a parallelogram ABCD, $\angle D = 135^{\circ}$, determine the measures of $\angle A$ and $\angle B$.

Solution

In parallelogram ABCD

$\angle D + \angle C = 180^\circ$ [Adjacent angles are supplementary]

$135^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 135^\circ = 45^\circ$

$\angle A = \angle C = 45^\circ$ [Opposite angles of a parallelogram are equal]

$\angle B = \angle D = 135^\circ$ [Opposite angles of a parallelogram are equal]

Q6

ABCD is a parallelogram in which ∠A = 70. Compute ∠B, ∠C and ∠D.

Solution

In parallelogram ABCD

$\angle A = 70^\circ$

$\angle A + \angle B = 180^\circ$ [Adjacent angles are supplementary]

$70^\circ + \angle B = 180^\circ$

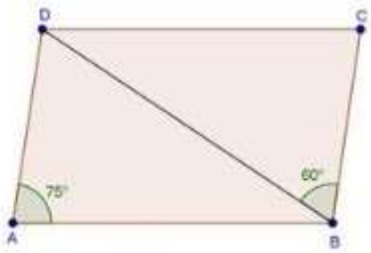
$\angle B = 180^\circ - 70^\circ = 110^\circ$

$\angle A = \angle C = 70^\circ$ [Opposite angles of a parallelogram are equal]

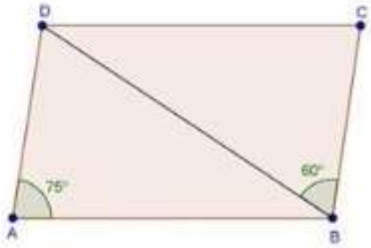
$\angle B = \angle D = 110^\circ$ [Opposite angles of a parallelogram are equal]

Q7

In fig., ABCD is a parallelogram in which ∠DAB = 75° and ∠DBC = 60°. Compute ∠CDB and ∠ADB.



Solution



To find $\angle CDB$ & $\angle ADB$

$\angle CBD = \angle ADB = 60°$ [Alternate interior angle $AD \parallel BC$ and BD is the transversal]

In parallelogram $ABCD$

$\angle A = \angle C = 75°$ [opposite angles of a parallelogram]

In ΔBDC

$\angle CBD + \angle C + \angle CDB = 180°$ [Angle sum property]

$\Rightarrow \quad 60° + 75° + \angle CDB = 180°$

$\Rightarrow \quad \angle CDB = 180° - (60° + 75°)$

$\angle CDB = 180° - 135° = 45°$

Hence, $\angle CDB = 45°$, $\angle ADB = 60°$

Q8

Which of the following statements are true(T) and which are false(F)?

(i) In a parallelogram, the diagonals are equal.

(ii) In a parallelogram, the diagonals bisect each other.

(iii) In a parallelogram, the diagonals intersect each other at right angles.

(iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.

(v) If all the angles of a quadrilateral are equal, it is a parallelogram.

(vi) If three sides of a quadrilateral are equal, it is a parallelogram.

(vii) If three angles of a quadrilateral are equal, it is a parallelogram.

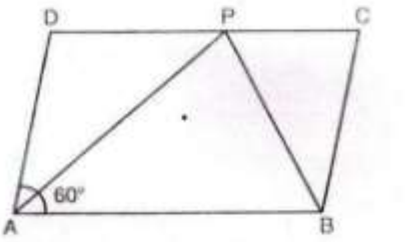
(viii) If all the sides of a quadrilateral are equal it is a parallelogram.

Solution

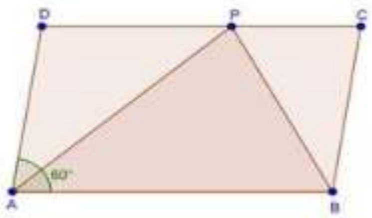
i. F
ii. T
iii. F
iv. F
v. T
vi. F
vii. F
viii. T

Q9

In fig., ABCD is a parallelogram in which $\angle A = 60°$. If the bisectors of $\angle A$ and $\angle b$ meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Solution



$\angle DAP = \angle PAB = 30°$ $\quad [\because AP \text{ bisects } \angle A]$

$\angle A + \angle B = 180°$ $\quad$ [Adjacent angles are supplementary]

$60° + \angle B = 180°$

$\angle B = 120°$

$\angle PBA = \angle PBC = 30°$ $\quad [\because BP \text{ bisect } \angle B]$

$\angle PAB = \angle APD = 30°$ $\quad$ [Alternate interior angles]

$\therefore AD = DP$ $\quad$ [Sides opposite to equal angle are equal in length]

Similarly

$\angle PBA = \angle BPC = 60°$ $\quad$ [Alternate interior angle]

$\therefore PC = BC$

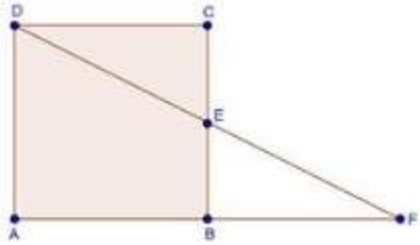
$DC = DP + PC$

$DC = AD + BC$ $\quad [\because DP = AD, PC = BC]$

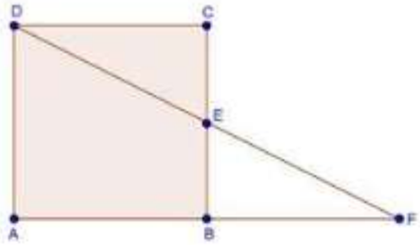
$DC = 2AD$ $\quad [\because AD = BC \text{ opposite sides of a parallelogram are equal}]$

Q10

In fig., ABCD is a parallelogram and E is the mid-point of side BC. IF DE and AB when produced meet at F, prove that AF = 2AB.



Solution



In ΔBEF and ΔCED

$\angle BEF = \angle CED$ [Vertically opposite angle]

$BE = CE$ [$\because$ E is the mid-point of BC]

$\angle EBF = \angle ECD$ [Alternate interior angles]

$\because$ $\Delta BEF \cong \Delta CED$ [A.S.A congruence]

$\therefore$ $BF = CD$ [C.P.C.T]

$AF = AB + BF$

$AF = AB + AB$ [$\because$ $BF = CD, CD = AB$]

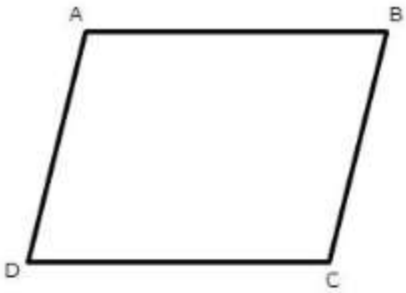
$AF = 2AB$

Exercise 13.3

Q1

In a parallelogram ABCD, determine sum of angles $\angle C$ and $\angle D$.

Solution



$\angle C$ and $\angle D$ are cosecutive interior angles on the same side of the transversal CD. Therefore,
$\angle C + \angle D = 180^o$

Q2

In a parallelogram ABCD, if $\angle B = 135°$, determine the measures of its other angles.

Solution

We have, $\angle B = 135°$

Since ABCD is a parallelogram

$\therefore \angle A = \angle C, \angle B = \angle D$ and $\angle A + \angle B = 180°$

$\Rightarrow \angle A + 135° = 180°$

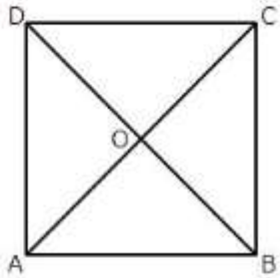
$\Rightarrow \angle A = 45°$

$\Rightarrow \angle A = \angle C = 45°$ and $\angle B = \angle D = 135°$

Q3

ABCD is a square. AC and BD intersect at O. State the measure of $\angle AOB$.

Solution

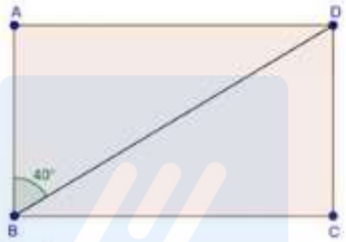


Since, diagonals of a square bisect each other at right angle. Therefore, $\angle AOB = 90^\circ$

Q4

ABCD is a rectangle with $\angle ABD = 40^\circ$. Determine $\angle DBC$.

Solution



We have,

$\angle ABC = 90^\circ$

$\Rightarrow \quad \angle ABD + \angle DBC = 90^\circ$

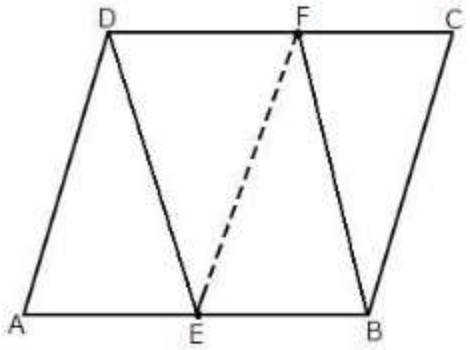
$\Rightarrow \quad 40^\circ + \angle DBC = 90^\circ$

$\Rightarrow \quad \angle DBC = 50^\circ$

Q5

The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Solution



Since $ABCD$ is a parallelogram. Therefore,

$AB \parallel DC$ and $AB = DC$

$\Rightarrow$ $EB \parallel DF$ and $\frac{1}{2}AB = \frac{1}{2}DC$

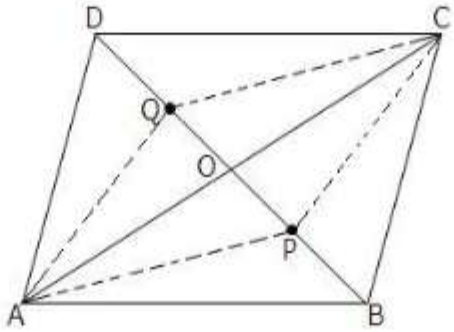
$\Rightarrow$ $EB \parallel DF$ and $EB = DF$

$\Rightarrow$ $EBFD$ is a parallelogram.

Q6

P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

Solution



Since, diagonals of a parallelogram bisect each other.

Therefore, $OA = OC$ and $OB = OD$.

Since, P and Q are points of trisection of BD.

$\therefore \quad BP = PQ = QD$

Now, $OB = OD$ and $BP = QD$

$\Rightarrow \quad OB - BP = OD - QD$

$\Rightarrow \quad OP = OQ$

Thus, in quadrilateral $APCQ$, we have

$OA = OC$ and $OP = OQ$

$\Rightarrow$ Diagonals of quadrilateral $APCQ$ bisect each other

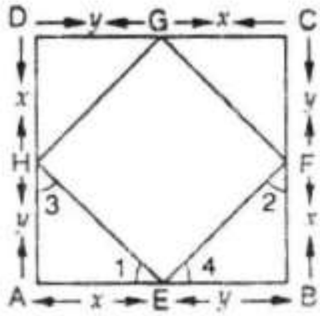
$\Rightarrow$ $APCQ$ is a parallelogram.

Hence, $AP \parallel CQ$.

Q7

ABCD is a square E, F, G and H are points on AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is square.

Solution



We have,

$AE = BF = CG = DH = x$ (say)

$\therefore$ $BE = CF = DG = AH = y$ (say)

In Δ's AEH and BEF, we have

$AE = BF$

$\angle A = \angle B$

and, $AH = BE$

So, by SAS congruence criterion, we have

$\Delta AEH \cong \Delta BFE$

$\Rightarrow$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But, $\angle 1 + \angle 3 = 90^\circ$ and $\angle 2 + \angle 4 = 90^\circ$

$\Rightarrow$ $\angle 1 + \angle 3 + \angle 2 + \angle 4 = 90^\circ + 90^\circ$

$\Rightarrow$ $\angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^\circ$

$\Rightarrow$ $2(\angle 1 + \angle 4) = 180^\circ$

$\Rightarrow$ $\angle 1 + \angle 4 = 90^\circ$

$\Rightarrow$ $HEF = 90^\circ$

Similarly, we have

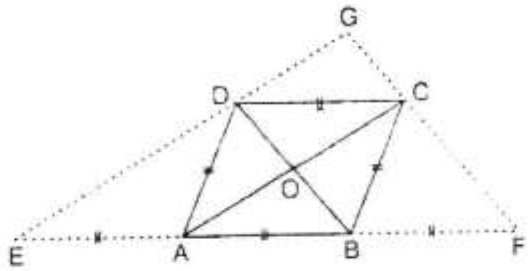
$\angle F = \angle G = \angle H = 90^\circ$

Hence, $EFGH$ is a square.

Q8

ABCD is a rhombus, EABF is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Solution



We know that the diagonals of a rhombus are perpendicular bisector of each other.

$\therefore$ $OA = OC, OB = OD, \angle AOD = \angle COD = 90^\circ$

And, $\angle AOB = \angle COB = 90^\circ$

In ΔBDE, A and O are mid-points of BE and BD respectively.

$\therefore$ $OA \parallel DE$

$\Rightarrow$ $OC \parallel DG$

In ΔCFA, B and O are mid-points AF and AC respectively

$\therefore$ $OB \parallel CF$

$\Rightarrow$ $OD \parallel GC$

Thus, in quadrilateral $DOCG$, we have

$OC \parallel DG$ and $OD \parallel GC$

$\Rightarrow$ $DOCG$ is a parallelogram.

$\therefore$ $\angle DGC = \angle DOC$

$\Rightarrow$ $\angle DGC = 90^\circ$

Q9

ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.

Solution

Draw a parallelogram ABCD with AC and BD intersecting at O.

Produce AD to E such that DE = DC.

Join EC and produce it to meet AB produced at F.

In $\triangle DCE$,

$\therefore \angle DCE = \angle DEC$ ...(1) (In a triangle, equal sides have equal angles opposite to them)

AB || CD (Opposite sides of the parallelogram are parallel)

$\therefore$ AF || CD (AB lies on AF)

AF || CD and EF is the transversal,

$\therefore \angle DCE = \angle BFC$ ...(2) (Pair of corresponding angles)

From (1) and (2), we get

$\angle DEC = \angle BFC$

In $\triangle AFE$,

$\angle AFE = \angle AEF$ ($\angle DEC = \angle BFC$)

$\therefore$ AE = AF (In a triangle, equal angles have equal sides opposite to them)

$\Rightarrow$ AD + DE = AB + BF

$\Rightarrow$ BC + AB = AB + BF ($\because$ AD = BC, DE = CD and CD = AB, AB = DE)

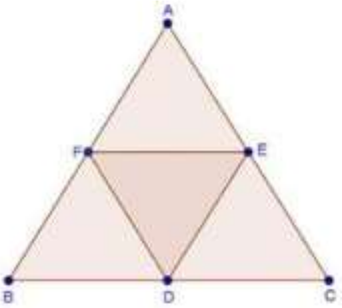
$\Rightarrow$ BC = BF

Exercise 13.4

Q1

In a $\Delta ABC, D, E$ and F are, respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Solution



$AB = 7$ cm, $BC = 8$ cm, $AC = 9$ cm

In ΔABC

$\therefore$ F & E are the mid-points of AB and AC

$\therefore$ $EF = \frac{1}{2}BC$ [Mid-points theorem]

Similarly

$DF = \frac{1}{2}AC, DE = \frac{1}{2}AB$

Perimeter of $\Delta DEF = DE + EF + DF$

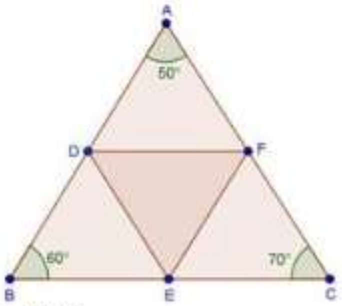
$$= \frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$$
$$= \frac{1}{2}\times 7 + \frac{1}{2}\times 8 + \frac{1}{2}\times 9$$
$$= 3.5 + 4 + 4.5 = 12 \text{ cm}$$

$\therefore$ P of $\Delta DEF = 12$ cm.

Q2

in a triangle $\angle ABC, \angle A = 50°, \angle B = 60°$ and $\angle C = 70°$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution



In ΔABC

D & E are mid-points of AB and BC

$\therefore \quad DE \parallel AC, DE = \frac{1}{2} AC$ [By mid-point theorem]

$DE = \frac{1}{2} AC = CF$ [$\because$ F is the mid-point of AC]

In quadrilateral $DECF$

$DE \parallel AC, DE = CF$

Hence $DECF$ is a parallelogram

$\therefore \quad \angle C = \angle D = 70°$ [Opposite angles of a parallelogram]

Similarly,

$BEFD$ is a parallelogram, $\angle B = \angle F = 60°$

$ADEF$ is a parallelogram, $\angle A = \angle E = 50°$

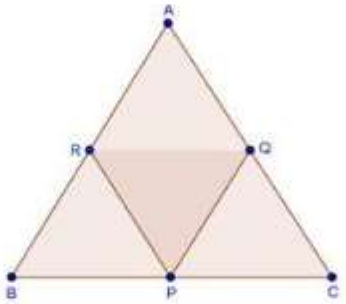
$\therefore$ Angles of ΔDEF

$\angle D = 70°, \angle E = 50°, \angle F = 60°$

Q3

In a triangle, P, Q and R are the mid-points of sides BC, CA and AB respectively, If $AC = 21$ cm, $BC = 29$ cm and $AB = 30$ cm, find the perimeter of the quadrilateral $ARPQ$.

Solution



In ΔABC

R & P are mid-points of AB and BC

$\therefore$ $RP \parallel AC$, $RP = \frac{1}{2}AC$ [By mid-point theorem]

In quadrilateral $ARPQ$

$RP \parallel AQ$, $RP = AQ$

$\therefore$ $ARPQ$ is a parallelogram [A pair of side is parallel and equal]

$AR = \frac{1}{2}AB = \frac{1}{2}\times 30 = 15$ cm

$\Rightarrow$ $AR = QP = 15$ [opposite sides are equal]

$RP = \frac{1}{2}AC = \frac{1}{2}\times 21 = 10.5$ cm

$\Rightarrow$ $RP = AQ = 10.5$ [Opposite sides are equal]

Now,

Perimeter of $ARPQ = AR + QP + RP + AQ$

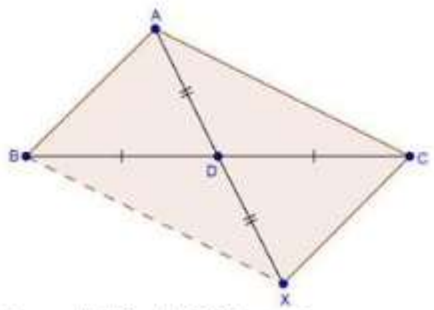
$= 15 + 15 + 10.5 + 10.5$

$= 51$ cm

Q4

In ΔABC median AD is produced to X such that $AD = DX$. Prove that $ABXC$ is a parallelogram.

Solution



In quadrilateral $ABXC$, we have

$AD = DX$ [Given]

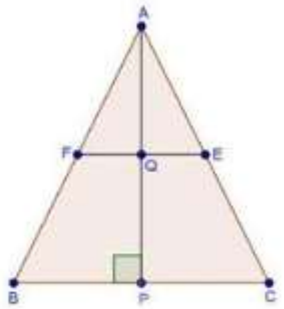
$BD = DC$ [Given]

So, diagonals AX and BC bisect each other. Therefore, $ABXC$ is a parallelogram.

Q5

In ΔABC E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that $AQ = QP$.

Solution



In ΔABC

F and E are mid-points of AB & AC

$\therefore$ $FE \parallel BC, FE = \frac{1}{2}BC$ [By mid-point theorem]

In ΔABP

F is the mid-point of AB and $FQ \parallel BP$ [$\because FE \parallel BC$]

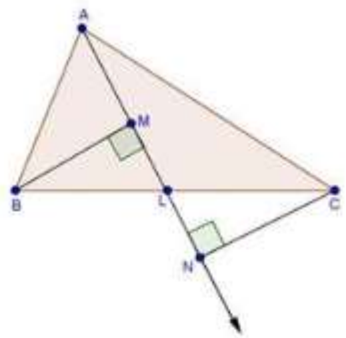
$\therefore$ Q is the mid-point of AP [By converse of mid-point theorem]

Hence, $AQ = QP$

Q6

In ΔABC BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that $ML = NL$.

Solution



In ΔBLM and ΔCLN

$\angle BML = \angle CNL = 90°$ [Given]

$BL = CL$ [L is the mid-point of BC]

$\angle MLB = \angle NLC$ [Vertically opposite angle]

$\therefore$ $\Delta BLM \cong \Delta CLN$ [$A.A.S$]

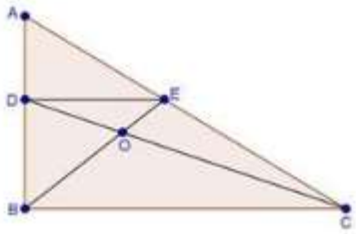
$\therefore$ $LM = LN$ [Corresponding parts of congrent triangles]

Q7

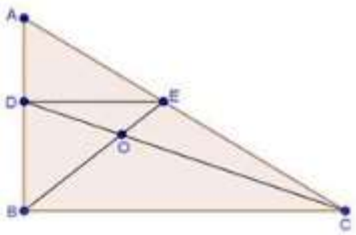
In fig., triangle ABC is right-angled at B. Give that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC

(ii) The area of ΔADE.



Solution



In right $\Delta ABC, \angle B = 90^\circ$

$$AC^2 = AB^2 + BC^2 \quad \text{[By pythagoras theorem]}$$
$$\Rightarrow 15^2 = 9^2 + BC^2$$
$$\Rightarrow 225 - 81 = BC^2$$
$$\Rightarrow 144 = BC^2$$
$$\Rightarrow BC = \sqrt{144} = 12 \text{ cm}$$

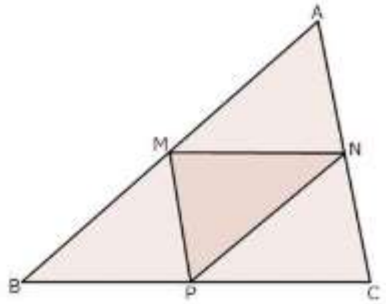
In ΔABC

D and E are mid-points of AB and AC

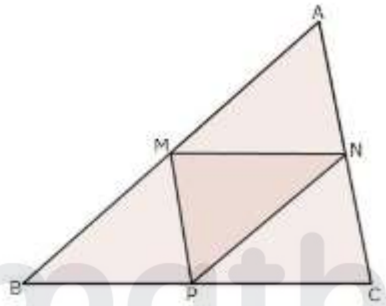
$$\therefore \quad DE \parallel BC,\ DE = \frac{1}{2}BC \quad \text{[By mid-point theorem]}$$
$$AD = DB = \frac{AB}{2} = \frac{9}{2} = 4.5 \text{ cm} \quad [\because D \text{ is the mid-point of } AB]$$
$$DE = \frac{BC}{2} = \frac{12}{2} = 6 \text{ cm}$$
$$\text{Area of } \Delta ADE = \frac{1}{2} \times AD \times DE$$
$$= \frac{1}{2} \times 4.5 \times 6 = 4.5 \times 3 = 13.5 \text{ cm}^2.$$

Q8

In fig., M, N, and P are the mid-points of AB, AC and BC respectively. If MN = 3 cm, Np = 3.5 cm and MP = 2.5 cm, Calculate BC, AB and AC.



Solution



Given, $MN = 3$ cm, $NP = 3.5$ cm and $MP = 2.5$ cm

To find, BC, AB and AC

In ΔABC

M and N are mid-points of AB and AC

$\therefore \quad MN = \frac{1}{2}BC,\ MN \parallel BC$ [By mid-point theorem]

$\Rightarrow \quad 3 = \frac{1}{2}BC$

$\Rightarrow \quad 3 \times 2 = BC$

$\Rightarrow \quad BC = 6$ cm

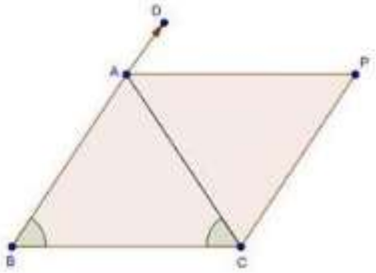
Similarly,

$AC = 2MP = 2 \times 2.5 = 5$ cm

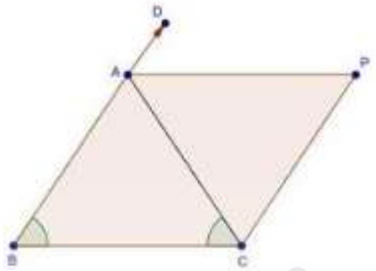
$AB = 2NP = 2 \times 3.5 = 7$ cm

Q9

In fig., AB = AC and CP ∥ BA and AP is the bisector of exterior ∠CAD of ΔABC. Prove that (i) ∠PAC = ∠BCA (ii) ABCP is a parallelogram.



Solution



Given,

$AB = AC$ and $CP \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of ΔABC.

To prove:

(i) $\angle PAC = \angle BCA$

(ii) $ABCP$ is a parallelogram

Proof:

(i) We have,

$AB = AC$

$\Rightarrow \quad \angle ACB = \angle ABC$ [Opposite angles of equal sides of triangle are equal]

Now,

$\angle CAD = \angle ABC + \angle ACB$

$\Rightarrow \quad \angle PAC + \angle PAD = 2\angle ACB$

$\Rightarrow \quad 2\angle PAC = 2\angle ACB \quad [\because \angle PAC = \angle PAD]$

$\Rightarrow \quad \angle PAC = \angle ACB$

$\Rightarrow \quad \angle PAC = \angle BCA$

(ii) Now,

$\angle PAC = \angle BCA$

$\Rightarrow \quad AP \parallel BC$

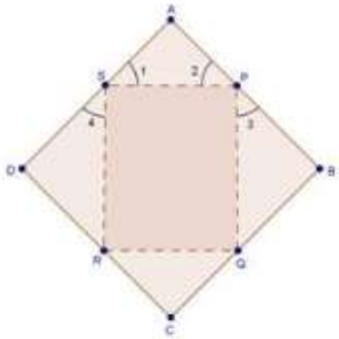
and, $\quad CP \parallel BA \quad$ [Given]

$\therefore$ $ABCP$ is a parallelogram.

Q10

$ABCD$ is a kite having $AB = AD$ and $BC = CD$. Prove that the figure formed by joining the mid-points of the sides, in order, is a rectangle.

Solution



Given,

A kite $ABCD$ having $AB = AD$ and $BC = CD$. P, Q, R, S are the mid-points of sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove:

$PQRS$ is a rectangle.

Proof: In ΔABC, P and Q are the mid-points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ —(i)

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore$ $RS \parallel AC$ and $RS = \frac{1}{2}AC$ —(ii)

From (i) and (ii), we have

$PQ \parallel RS$ and $PQ = RS$.

Thus, in quadrilateral $PQRS$, a pair of opposite sides are equal and parallel.
So, $PQRS$ is a parallelogram. Now, we shall prove that one angle of parallelogram $PQRS$ is a right angle.

Since, $AB = AD$

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}AD$

$\Rightarrow$ $AP = AS$ —(iii) $\left[\because P \text{ and } S \text{ are the mid-points of B and } AD \text{ respectively}\right]$

$\Rightarrow$ $\angle 1 = \angle 2$ —(iv)

Now, in ΔPBQ and ΔSDR, we have

$PB = SD$ $\left[\begin{array}{l} \because AD = AB \\ \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB \\ \therefore\ PB = SD \end{array}\right]$

$BQ = DR$ $[\because BC = DC]$

and, $PQ = SR$ $\left[\begin{array}{l} \because PQRS \text{ is parallelogram} \\ \therefore PQ = SR \end{array}\right]$

So, by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SDR$

$\Rightarrow$ $\angle 3 = \angle 4$ $[C.P.C.T]$

Now,

$\angle 3 + \angle SPQ + \angle 2 = 180°$

and $\angle 1 + \angle PSR + \angle 4 = 180°$

$\therefore$ $\angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow$ $\angle SPQ = \angle PSR$ $[\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively.

$\therefore$ $\angle SPQ + \angle PSR = 180°$

$\Rightarrow$ $2\angle SPQ = 180°$ $[\because \angle SPQ = \angle PSR]$

$\Rightarrow$ $\angle SPQ = 90°$

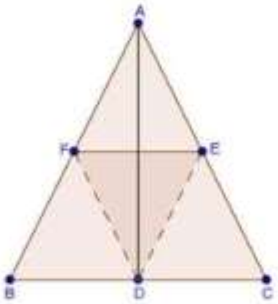
Thus, $PQRS$ is a parallelogram such that $\angle SPQ = 90°$

Hence, $PQRS$ is a rectangle.

Q11

Let ABC be an isosceles triangle in which $AB = AC$. If D, E, F be the mid-points of the sides BC, CA and AB respectively, show that the segment AD and EF bisect each other at right angles.

Solution



Since D, E and F are the mid-points of sides BC, CA and AB respectively.

$\therefore$ $AB \parallel DE$ and $AC \parallel FD$

$\Rightarrow$ $AF \parallel DE$ and $AE \parallel FD$

$\Rightarrow$ $AFDE$ is a parallelogram

$\Rightarrow$ $AF = DE$ and $AE = DF$

$\Rightarrow$ $\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

$\Rightarrow$ $DE = DF$ $[\because AB = AC]$

$\Rightarrow$ $AE = AF = DE = DF$

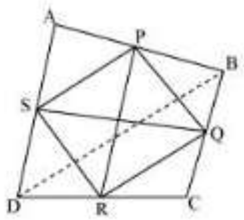
$\Rightarrow$ $AEDF$ is a rhombus

$\Rightarrow$ AD and FE bisect each other at right angle.

Q12

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution



Let ABCD is a quadrilateral in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.
Join PQ, QR, RS, SP and BD.
In Δ ABD, S and P are mid points of AD and AB respectively.
So, By using mid-point theorem, we can say that

SP || BD and SP = $\frac{1}{2}$ BD ... (1)

Similarly in Δ BCD

QR || BD and QR = $\frac{1}{2}$ BD ... (2)

From equations (1) and (2), we have
SP || QR and SP = QR
As in quadrilateral SPQR one pair of opposite sides are equal and parallel to each other.
So, SPQR is a parallelogram.
Since, diagonals of a parallelogram bisect each other.
Hence, PR and QS bisect each other.

Q13

Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles traingle is ______.

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is ______ .

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is ______ .

Solution

(i) isosceles

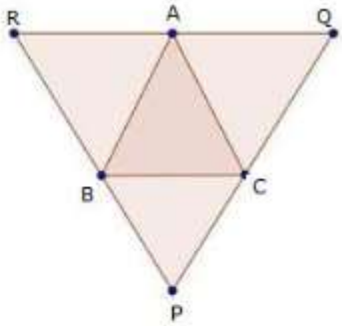
(ii) right triangle

(iii) parallelogram

Q14

ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Solution



Clearly, $ABCQ$ and $ARBC$ are parallelograms.

$\therefore \quad BC = AQ$ and $BC = AR$

$\Rightarrow \quad AQ = AR$

$\Rightarrow \quad A$ is the mid-point of QR.

Similarly, B and C are the mid-points of PR and PQ respectively.

$\therefore \quad AB = \frac{1}{2}PQ, BC = \frac{1}{2}QR$ and $CA = \frac{1}{2}PR$

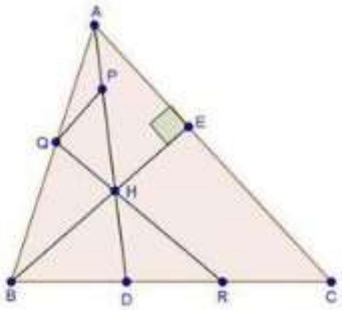
$\Rightarrow \quad PQ = 2AB, QR = 2BC$ and $PR = 2CA$

$\Rightarrow \quad PQ + QR + RP = 2(AB + BC + CA)$

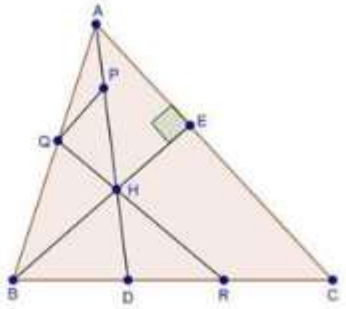
$\Rightarrow \quad$ Perimeter of $\Delta PQR = 2$ (Perimeter of ΔABC)

Q15

In fig., $BE \perp AC$. AD is any line from A to BC interesting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that $\angle PQR = 90°$.



Solution



Given,

$BE \perp AC$ and P, Q and R are respectively mid-point of AH, AB and BC.

To prove:

$\angle PQR = 90°$

Proof: In ΔABC, Q and R are the mid-points of AB and BC respectively.

$\therefore \quad QR \parallel AC \quad$ —(i)

In ΔABH, Q and P are the mid-points of AB and AH respectively.

$\therefore \quad QP \parallel BH$

$\Rightarrow \quad QP \parallel BE \quad$ —(ii)

But, $AC \perp BE$. Therefore, from equation (i) and equation (ii) we have

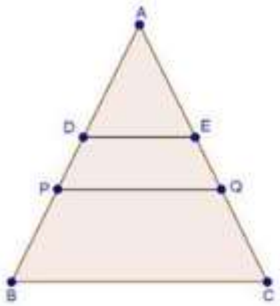
$QP \perp QR$

$\Rightarrow \quad \angle PQR = 90°$ Hence proved.

Q16

ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ and E is a point on AC such that $AE = \frac{1}{4}AC$. Prove that $DE = \frac{1}{4}BC$.

Solution



Let P and Q be the mid-points of AB and AC respectively.

Then, $PQ \parallel BC$ such that

$PQ = \frac{1}{2}BC \quad$ —(i)

In ΔAPQ, D and E are mid-points of AP and AQ are respectively.

$\therefore \quad DE \parallel PQ$ and $DE = \frac{1}{2}PQ \quad$ —(ii)

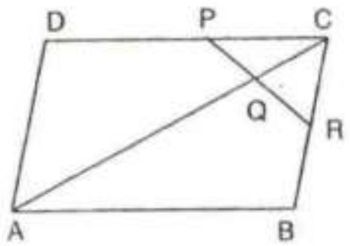
From equation (i) and equation (ii), we get

$$DE = \frac{1}{2}PQ = \frac{1}{2}\left[\frac{1}{2}BC\right]$$

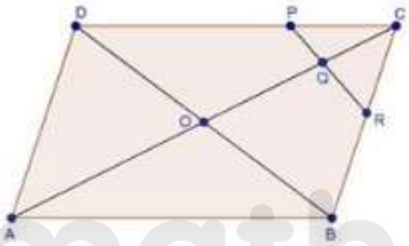
$\Rightarrow \quad DE = \frac{1}{4}BC$ Hence proved.

Q17

In fig., ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that CQ = (1/4)AC. If PQ produced meets BC at R, prove that R is a mid-point of BC.



Solution



Join B and D. Suppose AC and BD cut at O. Then,

$$OC = \frac{1}{2}AC$$

Now,

$$CQ = \frac{1}{4}AC$$

$$\Rightarrow \quad CQ = \frac{1}{2}\left[\frac{1}{2}AC\right]$$

$$= \frac{1}{2} \times OC \qquad \left[\because OC = \frac{1}{2}AC\right]$$

In ΔDCO, P and Q are mid-points of DC and OC respectively.

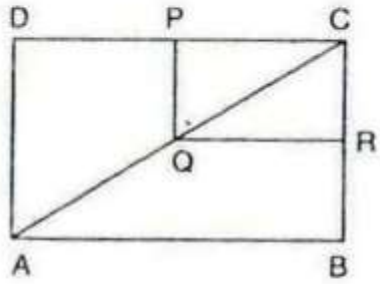
$\therefore \quad PQ \parallel DO$

Also, in ΔCOB, Q is mid-point of OC and $QR \parallel OB$

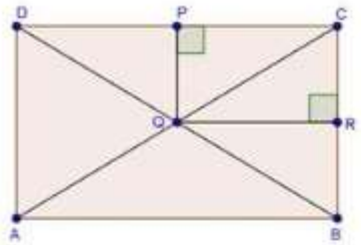
$\therefore$ R is the mid-point of BC.

Q18

In fig., ABCD and PQRC are rectangle and Q is the mid-point of AC. Prove that
i. DP = PC ii. PR = (1/2) AC



Solution



(i) In $\Delta ADC, Q$ is the mid-point of AC such that $PQ \parallel AD$

$\therefore$ P is the mid-point of DC

$\Rightarrow$ $DP = PC$ [Using converse of mid-point theorem]

(ii) Similarly, R is the mid-point of BC.

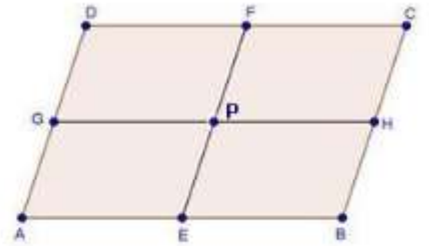
$\therefore$ $PR = \frac{1}{2} BD$

$\Rightarrow$ $PR = \frac{1}{2} AC$ [Diagonals of rectangle are equal, $\therefore$ BD = AC]

Q19

ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC and G, P and H respectively. Prove that GP = PH.

Solution



Since E and F are mid-points AB and CD respectively.

$\therefore \quad AE = BE = \frac{1}{2}AB$

and $\quad CF = DF = \frac{1}{2}CD$

But, $\quad AB = CD$

$\therefore \quad \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow \quad BE = CF$

Also, $\quad BE \parallel CF \qquad [\because AB \parallel CD]$

$\therefore \quad BEFC$ is a parallelogram.

$\Rightarrow \quad BC \parallel EF$ and $BE = PH \qquad$ —(i)

Now, $\quad BC \parallel EF$

$\Rightarrow \quad AD \parallel EF \qquad [\because BC \parallel AD$ as $ABCD$ is a parallelogram$]$

$\Rightarrow \quad AEFD$ is a parallelogram

$\Rightarrow \quad AE = GP \qquad$ —(ii)

But, E is the mid-point of AB.

$\therefore \quad AE = BE$

$\Rightarrow \quad GP = PH \qquad$ [Using (i) and (ii)]

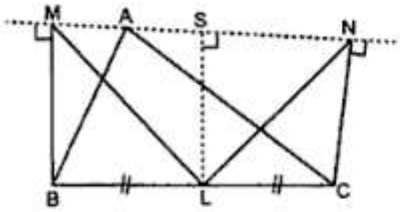
Q20

BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Solution

To prove: LM = LN

Draw LS perpendicular to line MN.



Therefore, the lines BM, LS and CN being the same perpendiculars, on line MN are parallel to each other.

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

In the drawn figure, MB and LS and NC are three parallel lines and the two transversal lines are MN and BC.

We have, BL = LC (As L is the given mid-point of BC)

$\therefore$ Using intercept theorem, we get,

MS = SN ...(1)

Now, In ΔMLS and ΔLSN

MS = SN using ...(1)

$\angle LSM = \angle LSN = 90^\circ$ $LS \perp MN$

and SL = LS common

$\therefore$ $\Delta MLS \cong \Delta LSN$ (SAS congruency criterion)

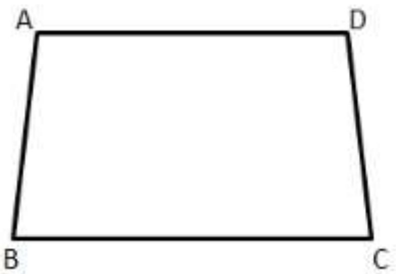
$\therefore$ LM = LN (CPCT)

Exercise 13.70

Q1

The opposite sides of a quadrilateral have
(a) no common point
(b) one common point
(c) two common points
(d) infinitely many common points

Solution

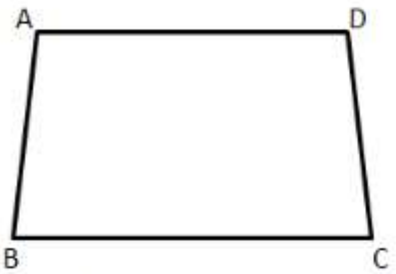


ABCD is a Quadrilateral.
The opposite sides AB and DC, AD and BC have no common point.
Hence, correct option is (a).

Q2

The consecutive sides of a quadrilateral have
(a) no common point
(b) one common point
(c) two common points
(d) infinitely many common points

Solution



Consecutive sides of a Quadrilateral ABCD are
AB and BC,
BC and CD,
CD and AD,
AD and AB,
which have only one point in common
i.e the joint point of their ends.
Hence, correct option is (b).

Exercise 13.71

Q1

PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases, PQRS is parallelogram?
(a) $\angle P = 100°, \angle Q = 80°, \angle R = 100°$
(b) $\angle P = 85°, \angle Q = 85°, \angle R = 95°$
(c) PQ = 7 cm, QR = 7 cm, RS = 8 cm, SP = 8 cm
(d) OP = 6.5 cm, OQ = 6.5 cm, OR = 5.2 cm, OS = 5.2 cm

Solution

In a parallelogram, opposite corner angles are equal and sum of adjacent angles = 180°
Hence, in quadrilateral PQRS,
$\Rightarrow \angle P = \angle R$ and $\angle Q = \angle S$
Also, $\angle P + \angle Q = \angle Q + \angle R = 180°$
Hence, if $\angle P = 100°$ and $\angle Q = 80°$, then
$\angle P + \angle Q = 100° + 80° = 180°$
Also, if $\angle Q = 80°$ and $\angle R = 100°$, then
$\angle Q + \angle R = 80° + 100° = 180°$
Hence, correct option is (a).

Q2

Which of the following quadrilateral is not a rhombus?
(a) All four sides are equal
(b) Diagonals bisect each other
(c) Diagonals bisect opposite angles
(d) One angle between the diagonals is 60°

Solution

For a rhombus, the angle between the diagonals is 90° and not 60°.
Hence, correct option is (d).

Q3

Diagonals necessarily bisect opposite angles in a
(a) rectangle
(b) parallelogram
(c) isosceles trapezium
(d) square

Solution

Diagonals necessarily bisect opposite angles in a square.
Hence, correct option is (d).

Q4

The two diagonals are equal in a
(a) parallelogram
(b) rhombus
(c) rectangle
(d) trapezium

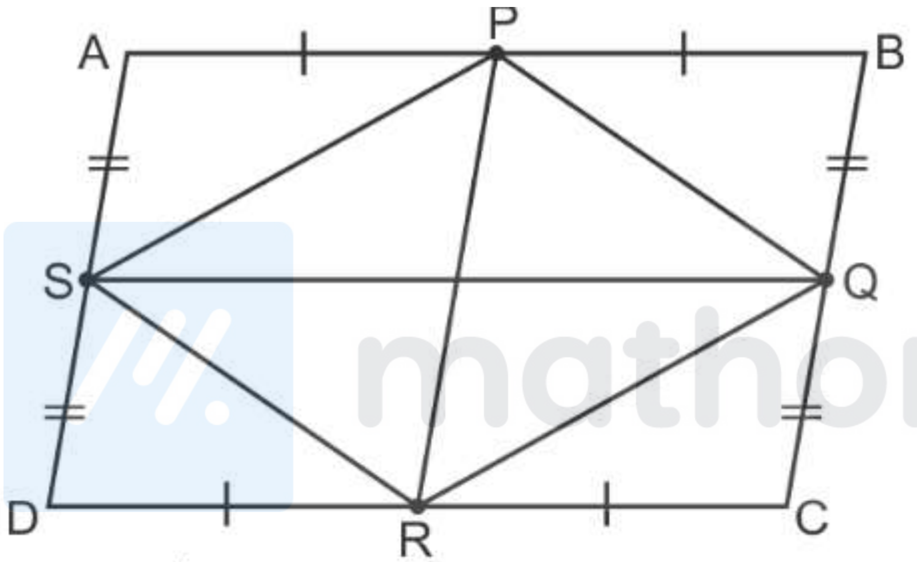
Solution

The two diagonals are equal in a rectangle (property).
Hence, correct option is (c).

Q5

We get a rhombus by joining the mid-points of the sides of a
(a) parallelogram
(b) rhombus
(c) rectangle
(d) triangle

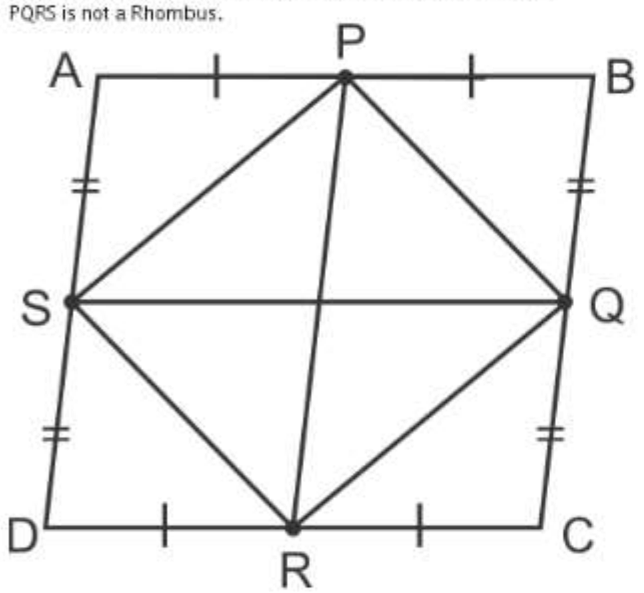
Solution



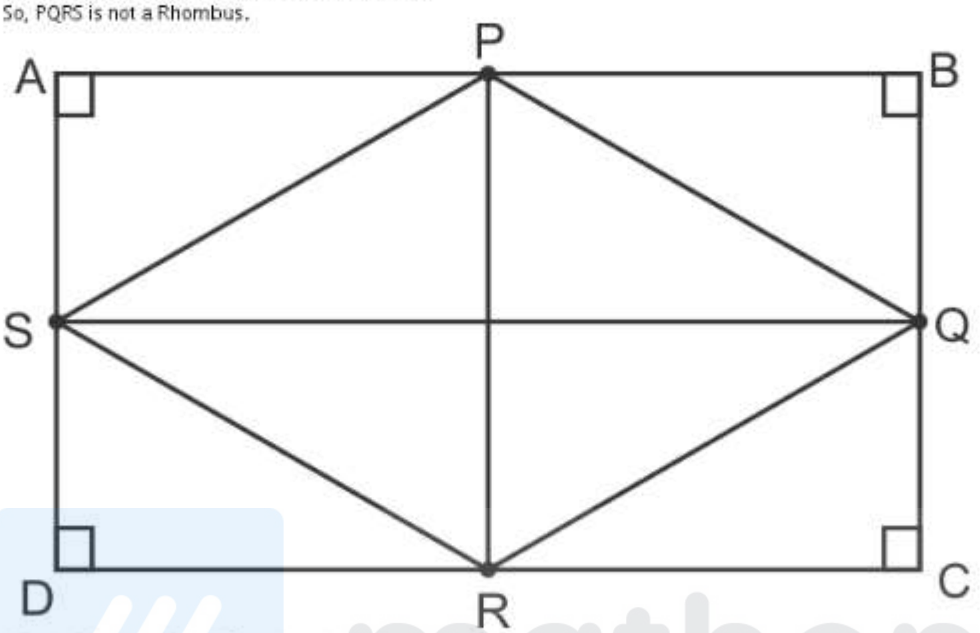
$PR \parallel AD \Rightarrow AB \not\perp AD$

$QS \parallel AB \Rightarrow PR \not\perp QS$

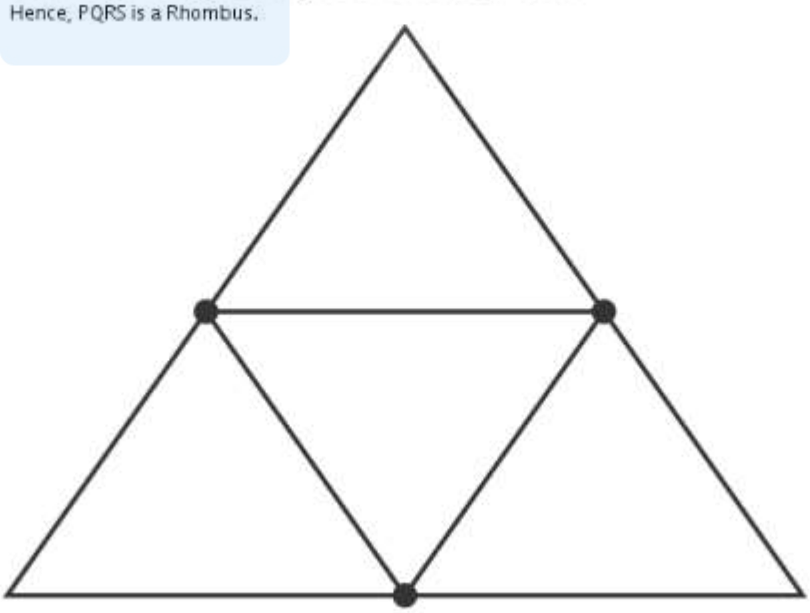
Since diagonals of PQRS are not making 90° between them,
PQRS is not a Rhombus.



P, Q, R and S are the mid – points,
PR and QS are diagonals of quadrilateral PQRS.
PR || AD, QS || AB
Because they are Formed by joning of mid – point of sides of Rhombus ABCD.
AD is not ⊥ to AB
⇒ PR will not be ⊥ to QS
i.e angle between diagonals PR & QS is not 90°.
So, PQRS is not a Rhombus.



PR and QS are making 90° with each – other
because PR || AD, QS || AB and AD ⊥ AB
So PR and QS are diagonals of PQRS and are ⊥ to each – other.
Hence, PQRS is a Rhombus.



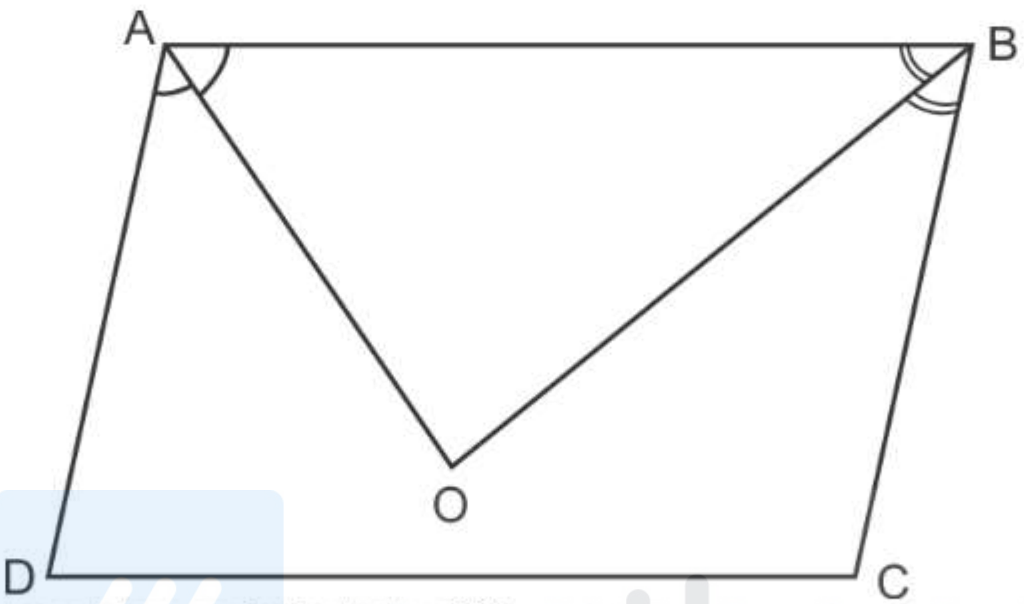
By joining the mid – points of sides of a triangle, no quadrilateral is formed.

hence, correct option is (c).

Q6

The bisectors of any two adjacent angles of a parallelogram intersect at
(a) 30°
(b) 45°
(c) 60°
(d) 90°

Solution



In a parallelogram, sum of adjacent angles = 180°

$\Rightarrow \angle A + \angle B = 180°$

$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} = 90°$ ....(1)

$\Rightarrow \angle OAB = \frac{\angle A}{2}$ and $\angle OBA = \frac{\angle B}{2}$

Thus, $\angle OAB + \angle OBA = 90°$ [From eq (1)]

$\Rightarrow \angle AOB = 180° - (\angle OAB + \angle OBA) = 180° - 90°$

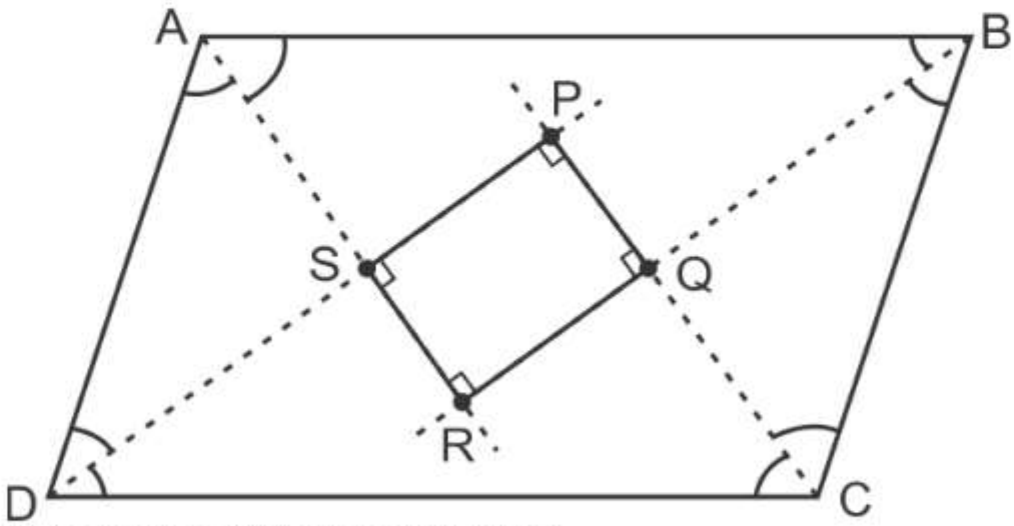
$\Rightarrow \angle AOB = 90°$

Hence, correct option is (d).

Q7

The bisectors of the angle of a parallelogram enclose a
(a) parallelogram
(b) rhombus
(c) rectangle
(d) square

Solution

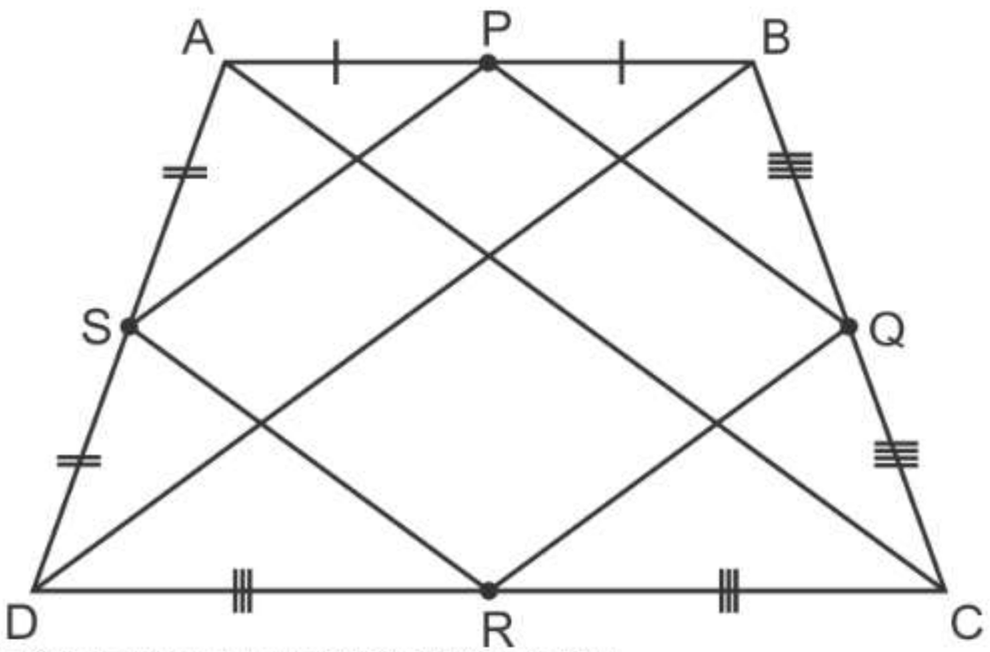


AR, BR, CP, DP are the bisectors of angles of parallelogram.
Because two bisectors of adjacent angles make 90° between them So PQRS is a Rectangle
Because DP and BR are acute angle bisectors so the distance between them PQ < PS (The distance between other two bisectors)
So PQ ≠ PS (So PQRS is not a square, but only a rectangle)
Hence, correct option is (c).

Q8

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a
(a) parallelogram
(b) rectangle
(c) square
(d) rhombus

Solution

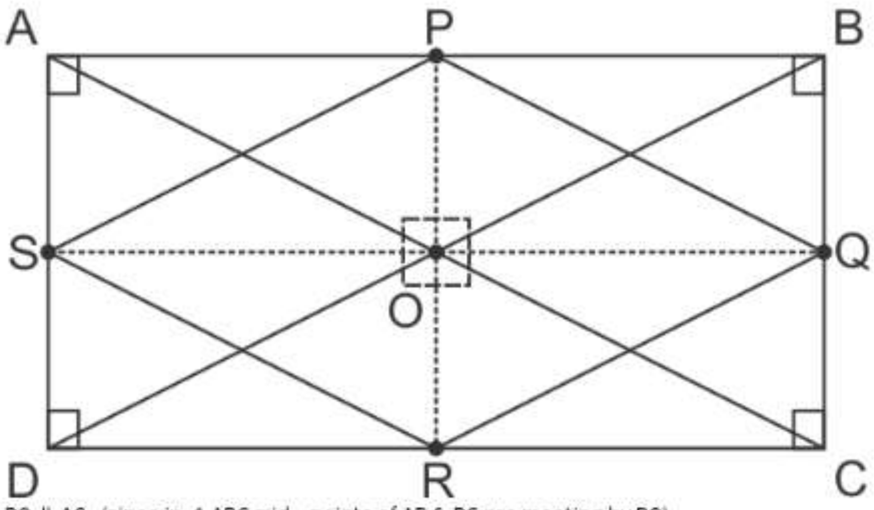


P, Q, R & S are the mid – points of AB, BC, CD & AD respectively.
Consider $\triangle ADB$,
If in a triangle, the mid – points of two sides are joint by a line then the line is parallel to the third side.
$\Rightarrow PS \parallel DB$ in $\triangle ADB$
Similiarly in $\triangle CDB$,
$RQ \parallel DB$
Hence $PS \parallel RQ$ ...(1)
Similarly in $\triangle ABC$ and $\triangle ADC$
$SR \parallel AC$, $PQ \parallel AC$
$\Rightarrow SR \parallel PQ$ ...(2)
From eq. (1) and (2), PQRS is a parallelogram.
Hence, correct option is (a).

Q9

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a
(a) square
(b) rhombus
(c) trapezium
(d) none of these

Solution

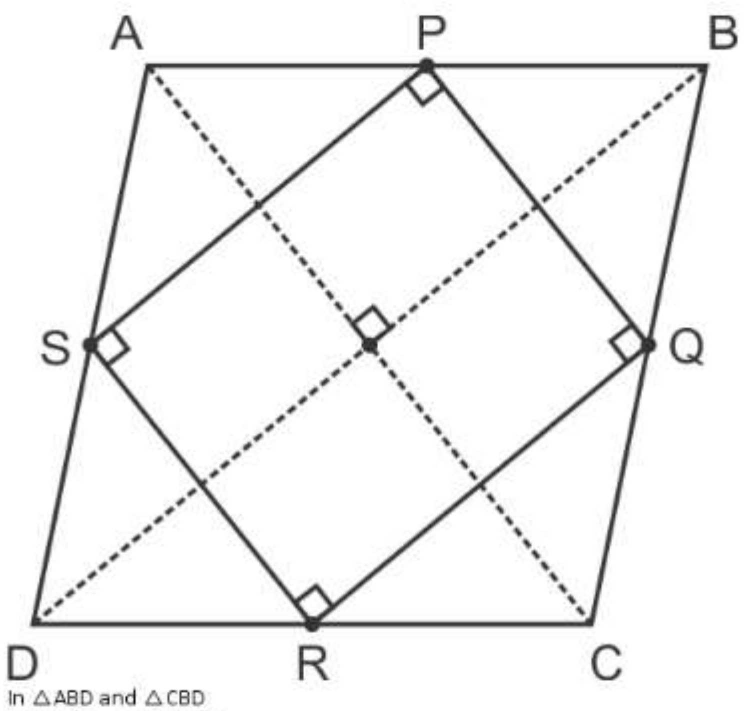


PQ || AC (since in $\triangle ABC$ mid – points of AB & BC are meeting by PQ)
Similiarly, SR || AC
$\Rightarrow$ PQ || SR
Now in $\triangle ABD$ and $\triangle CBD$,
PS || BD and QR || BD
$\Rightarrow$ PS || QR
Hence, PQRS is a parallelogram.
But PR $\perp$ QS
$\Rightarrow$ Diagonals cut at 90°
$\Rightarrow$ PQRS is a Rhomus
Hence, correct option is (b).

Q10

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a
(a) square
(b) rectangle
(c) trapezium
(d) none of these

Solution

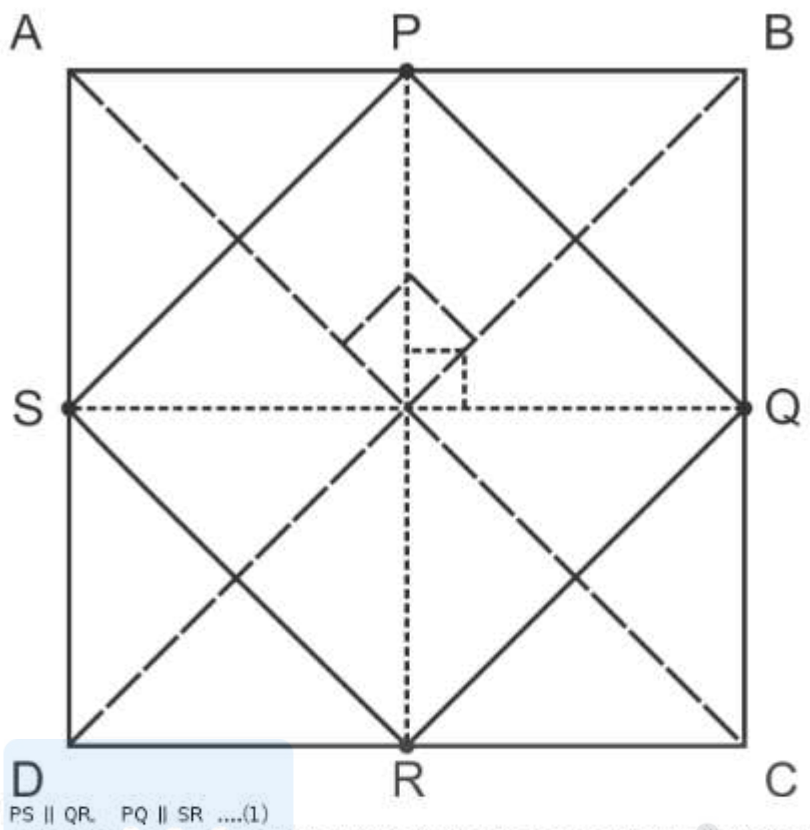


In $\triangle ABD$ and $\triangle CBD$
$PS \parallel BD$ and $QR \parallel BD$
{A line joining mid – points of two sides of $\triangle$ is parallel to third side}
$\Rightarrow PS \parallel QR$
Similiarly $PQ \parallel SR$
Because $SR \parallel AC$ and $QR \parallel BD$,
And angle between the diagonals of a Rhombus AC and BD $= 90°$,
Angle between SR and QR $= 90°$
$\Rightarrow$ PQRS is a Rectangle.
Hence, correct option is (b).

Q11

The figure formed by joining the mid-points of the adjacent sides of a square is a
(a) rhombus
(b) square
(c) rectangle
(d) parallelogram

Solution



PS || QR, PQ || SR ....(1)

{Because lines joining the midpoint of any two sides of a $\triangle$ are parallel to the third side}

AC $\perp$ BD & PR $\perp$ QS (From Figure)

SR || AC and QR || BD

AC $\perp$ BD

$\Rightarrow$ SR $\perp$ QR

Hence $\angle SRQ = 90°$ ....(2)

Also $\triangle APS \cong \triangle DSR$

$\Rightarrow$ PS = SR ...(3)

From equations (1), (2), (3)

PQRS is a square.

Hence, correct option is (b).

Q12

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

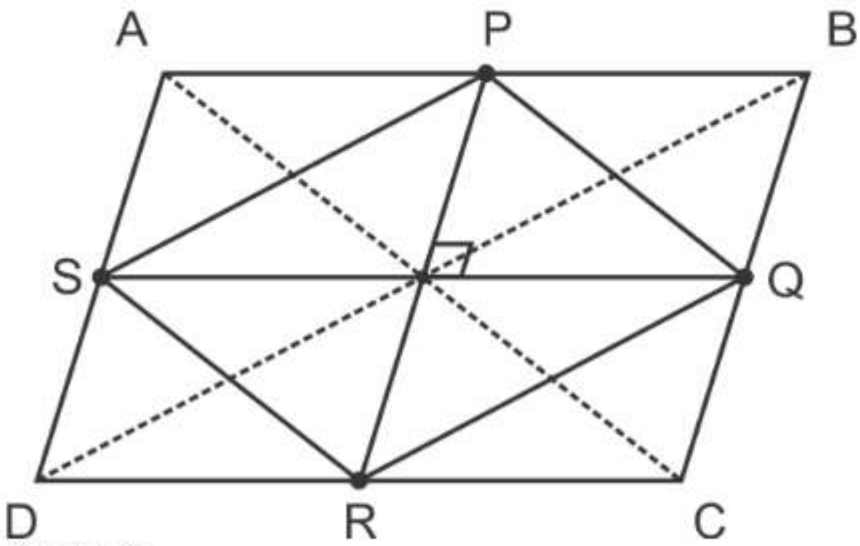
(a) rectangle

(b) parallelogram

(c) rhombus

(d) square

Solution

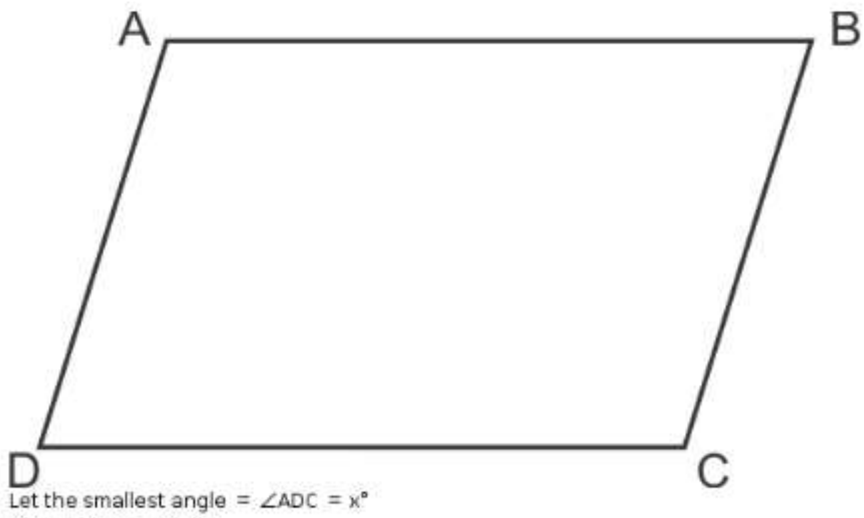


PQ || SR || AC
QR || PS || BD
{Because line joining the mid – points of two sides of $\triangle$ is || to third side}
Now because AC is not prependicular to BD in parallelogram,
$\Rightarrow$ SR is not perpendicular to QR
Also $\triangle ASP \not\cong \triangle DRS$
$\Rightarrow PS \neq SR$
$\Rightarrow$ PQRS is just a parallelogram.
Hence, correct option is (b).

Q13

If one angle of a parallelogram is 24° less than twice the smallest angle, then the measure of the largest angle of the parallelogram is
(a) 176°
(b) 68°
(c) 112°
(d) 102°

Solution



Let the smallest angle $= \angle ADC = x°$

Other angle $= \angle BCD$

$\Rightarrow \angle BCD = 2x° - 24°$

Also, $\angle ADC + \angle BCD = 180°$ (Sum of adjacent angles in $\|^{gram} = 180°$)

$\Rightarrow x° + 2x° - 24° = 180°$

$\Rightarrow 3x° = 204°$

$\Rightarrow x = 68°$

$\Rightarrow$ Largest angle $= \angle BCD = 2 \times 68° - 24° = 112°$

Hence, correct option is (c).

Q14

In a parallelogram ABCD, if $\angle DAB = 75°$ and $\angle DBC = 60°$, then $\angle BDC =$

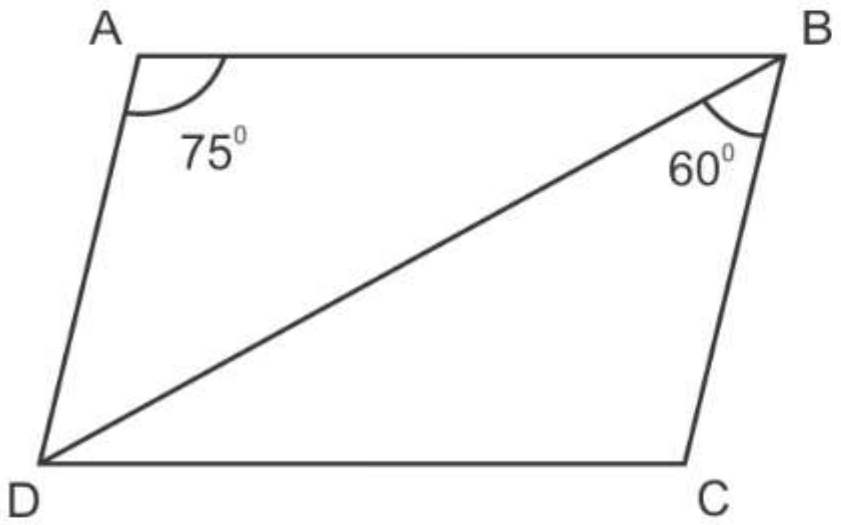
(a) 75°

(b) 60°

(c) 45°

(d) 55°

Solution



In parallelogram ABCD,

$\angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - 75° = 105°$

$\angle ADB = \angle DBC$ (Alternate angles)

$\Rightarrow \angle ADB = 60°$

$\angle BDC = \angle ADC - \angle ADB = 105° - 60° = 45°$

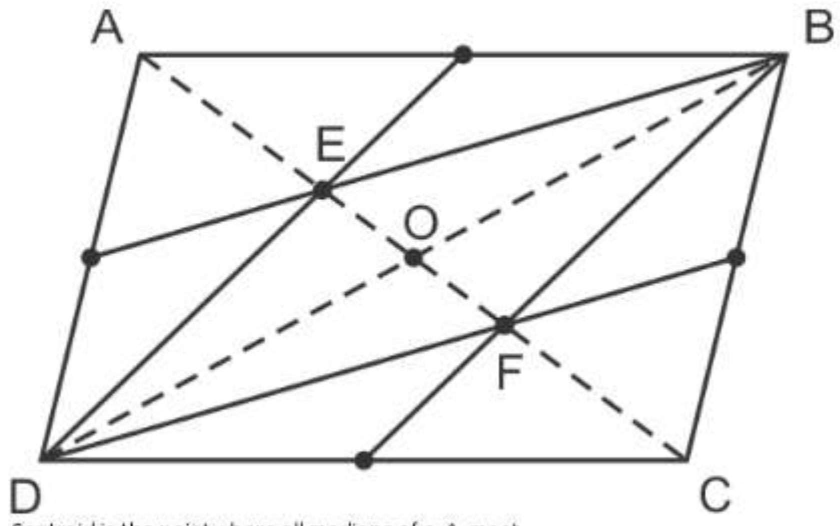
Hence, correct option is (c).

Q15

ABCD is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF =

(a) AE
(b) BE
(c) CE
(d) DE

Solution



Centroid is the point where all medians of a $\triangle$ meet.
In $\triangle ABD$, E is the centroid,
and in $\triangle BCD$, F is the centroid.
By the property of centroid, centroid divides a median in 2 : 1
So from figure,

$\frac{AE}{EO} = \frac{2}{1} \Rightarrow EO = \frac{AE}{2}$ ...(1)

Also $\frac{CF}{FO} = \frac{2}{1} \Rightarrow FO = \frac{CF}{2}$ ...(2)

Because AC is a diagonal of a paralleogram, O is its midpoint.

$\Rightarrow OA = OC$

$\Rightarrow AE = CF$

Adding equations (1) & (2),

$EO + FO = \frac{AE + CF}{2} = \frac{\cancel{2}AE}{\cancel{2}}$

$\Rightarrow EF = \frac{\cancel{2}AE}{\cancel{2}}$

$\Rightarrow EF = AE$

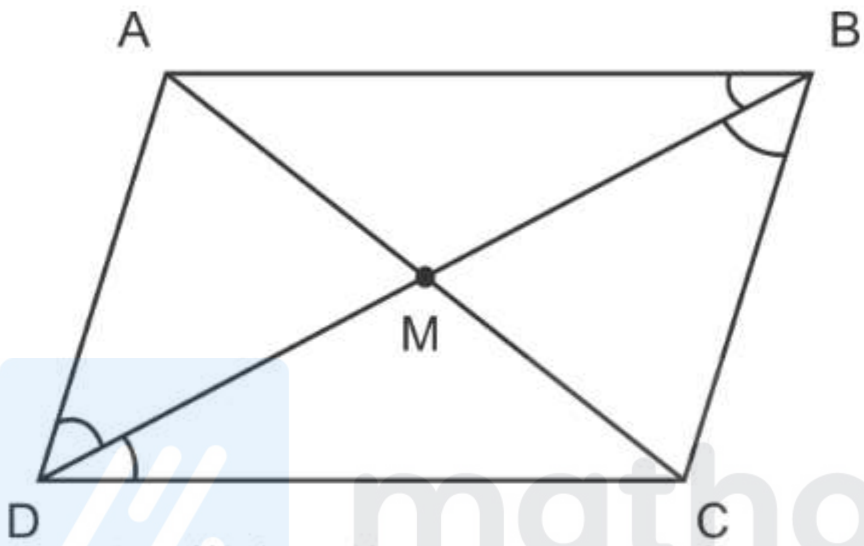
Hence, correct option is (a).

Exercise 13.72

Q1

ABCD is a parallelogram. M is the mid – point of BD and BM bisects $\angle B$. Then, $\angle AMB$ =
(a) 45°
(b) 60°
(c) 90°
(d) 75°

Solution

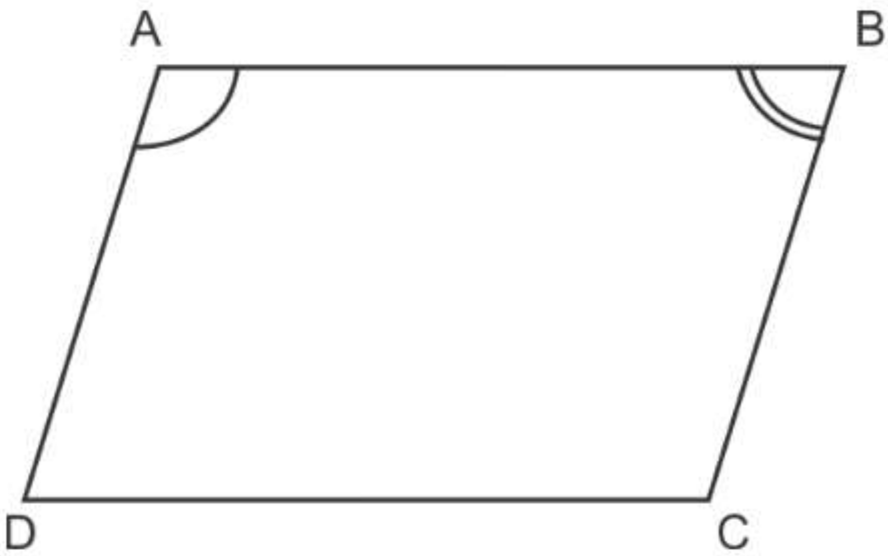


$\angle ABM = \angle CBM$ ...(1) (BM bisects $\angle B$)
$\angle ABM = \angle MDC$ ...(2) (Alternate angles)
$\angle CBM = \angle ADM$ ....(3) (Alternate angles)
From equations (1), (2) & (3)
$\angle MDC = \angle ADM$ ....(4)
Now, consider $\triangle ABD$ & $\triangle CBD$
$\angle CBD = \angle ABD$ {From eq (1)}
DB = DB (Common)
$\angle ADB = \angle CDB$ {From eq (4)}
Hence, by ASA property,
$\triangle ABD \cong \triangle CBD$
$\Rightarrow AB = CB, AD = CD$
Hence, it becomes a Rhombus.
So, now diagonals of a Rhombus bisect each other at 90°.
$\Rightarrow \angle AMB = 90°$
Hence, correct option is (c).

Q2

If an angle of a parallelogram is is two-third of its adjacent angle, the smallest angle of the parallelogram is
(a) 108°
(b) 54°
(c) 72°
(d) 81°

Solution



Let ABCD be a parallelogram and $\angle A = \frac{2}{3}\angle B$

Also, $\angle A + \angle B = 180°$ (adjacent angles in a Parallelogram are supplementry)

$\Rightarrow \frac{2}{3}\angle B + \angle B = 180°$

$\Rightarrow \angle B = 108°$ and $\angle A = 72°$

$\Rightarrow$ Smallest angle is 72°.

Hence, correct option is (c).

Q3

If the degree measures of the angles of quadrilateral are 4x, 7x, 9x and 10x, what is the sum of the measures of the smallest angle and largest angle?
(a) 140°
(b) 150°
(c) 168°
(d) 180°

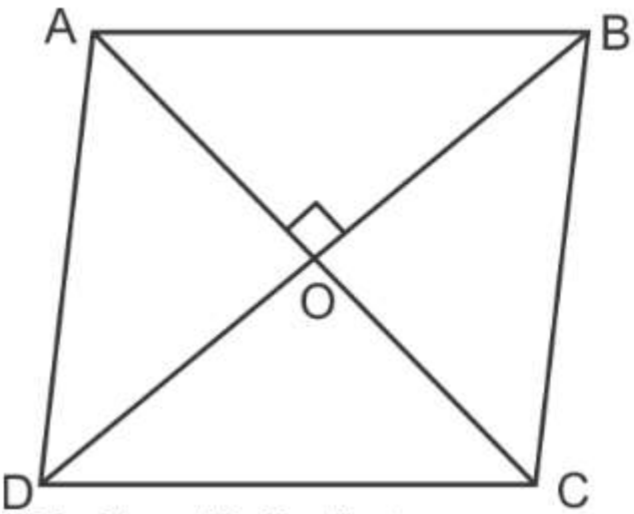
Solution

Sum of all angles of a Quadrilateral = 360°
$4x + 7x + 9x + 10x = 360°$
$30x = 360°$
$x = 12°$
So, sum of smallest and largest angle,
i.e. $4x + 10x = 14x = 14 \times 12 = 168°$
Hence, correct option is (c).

Q4

If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to
(a) 16 cm
(b) 15 cm
(c) 20 cm
(d) 17 cm

Solution



Let BD = 24 cm and AC = 18 cm (Given)

Now, $AO = \frac{AC}{2} = \frac{18}{2} = 9$ cm and $BO = \frac{BD}{2} = \frac{24}{2} = 12$ cm

Now, $AB = \sqrt{(AO)^2 + (BO)^2}$ (Diagonals make 90° between them)

$= \sqrt{9^2 + 12^2}$

$= \sqrt{81 + 144}$

$= \sqrt{225}$

$\Rightarrow AB = 15$ cm

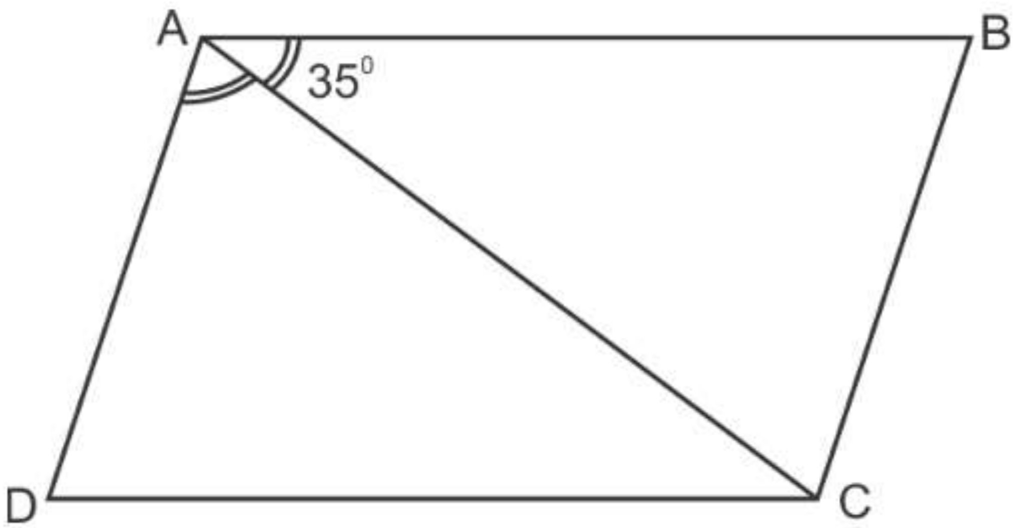
Hence, correct option is (b).

Q5

ABCD is a parallelogram in which diagonal AC bisects $\angle BAD$. If $\angle BAC = 35°$, then $\angle ABC$ =

(a) 70°
(b) 110°
(c) 90°
(d) 120°

Solution



AC bisects $\angle DAB$.

$\Rightarrow \angle DAC = \angle BAC = 35°$

$\Rightarrow \angle BAD = 2 \times 35° = 70°$

$\angle A + \angle B = 180°$ (Sum of any two adjacent angles in Parallelogram = 180°)

$\Rightarrow \angle B = \angle ABC = 180° - \angle BAD = 180° - 70° = 110°$

Hence, correct option is (b).

Q6

In a rhombus ABCD, if $\angle ACB = 40°$, then $\angle ADB =$

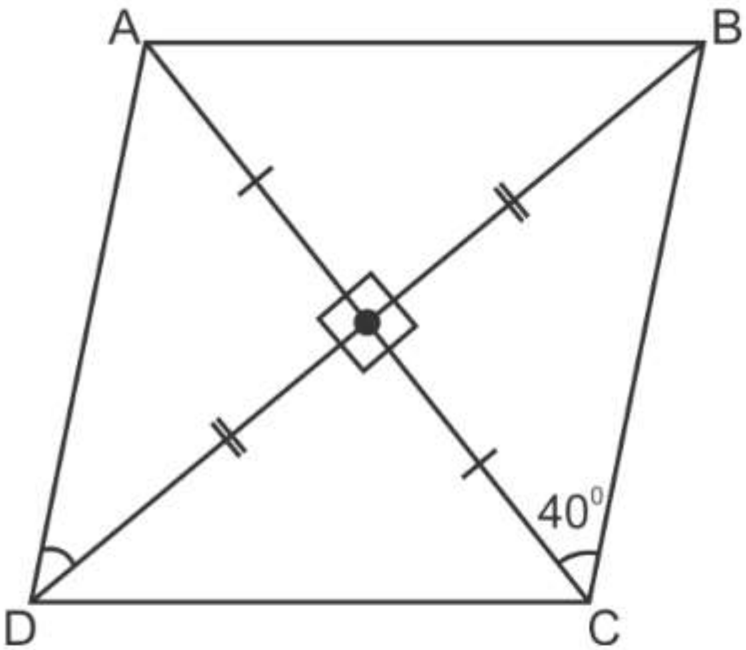
(a) 70°

(b) 45°

(c) 50°

(d) 60°

Solution

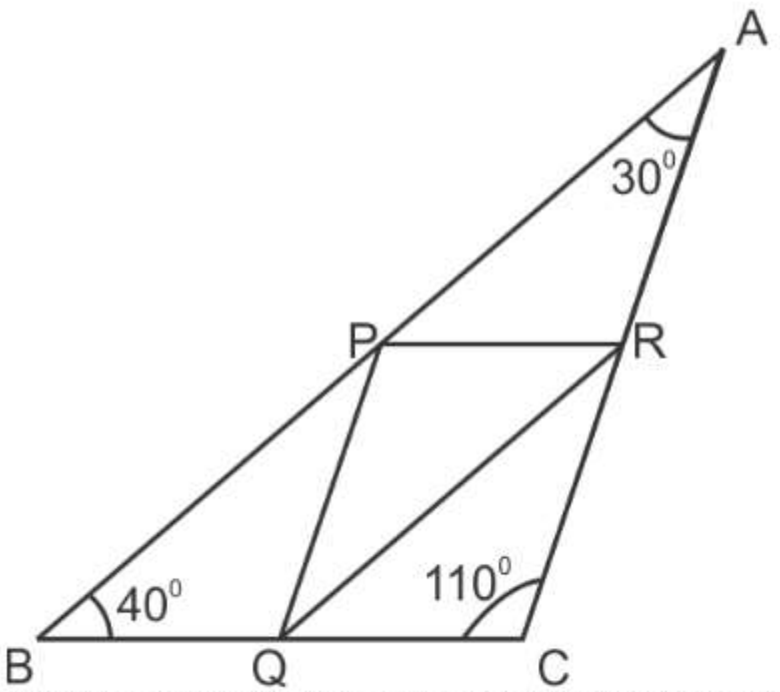


Consider $\triangle AOD$ & $\triangle COB$
$\angle AOD = \angle COB = 90°$
$AD = BC$ (Sides of Rhombus)
$AO = CO$ (Diagonals bisects each other)
So by RHS property, $\triangle AOD \cong \triangle COB$
$\Rightarrow \angle OAD = \angle OCB = 40°$
In $\triangle AOD$,
$\angle ADB = \angle ADO = 180° - 90° - 40° = 50°$
Hence, correct option is (c).

Q7

In $\triangle ABC$, $\angle A = 30°$, $\angle B = 40°$ and $\angle C = 110°$. The angles of the triangle formed by joining the mid – points of the sides of this triangle are
(a) $70°, 70°, 40°$
(b) $60°, 40°, 80°$
(c) $30°, 40°, 110°$
(d) $60°, 70°, 50°$

Solution

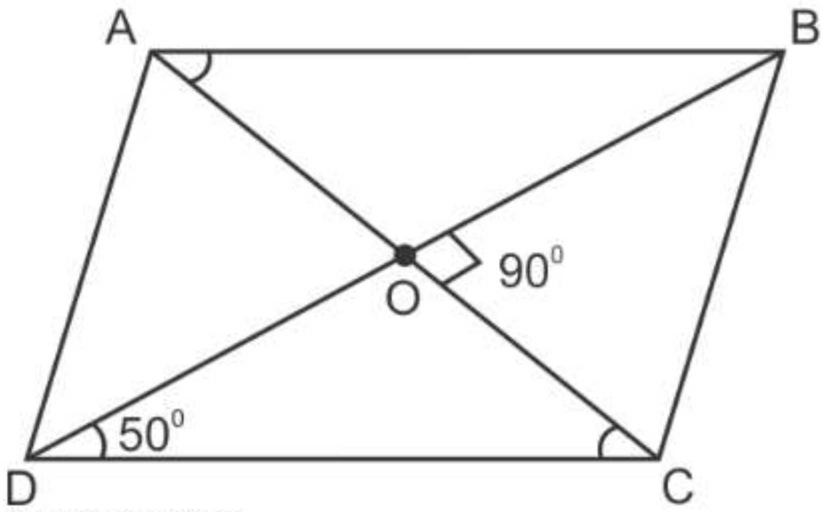


If in any triangle, all the mid – points (of each sides) are joined to form a triangle, then that triangle is similian to a parent triangle.
i.e. $\triangle QPR \sim \triangle ABC$
So angles of $\triangle PQR$ will be same as angles of $\triangle ABC$.
Thus, angles are $30°$, $40°$, $110°$.
Hence, correct option is (c).

Q8

The diagonals of a parallelogram ABCD intersect at O. If $\angle BOC = 90°$ and $\angle BDC = 50°$, then $\angle OAB =$
(a) $40°$
(b) $50°$
(c) $10°$
(d) $90°$

Solution

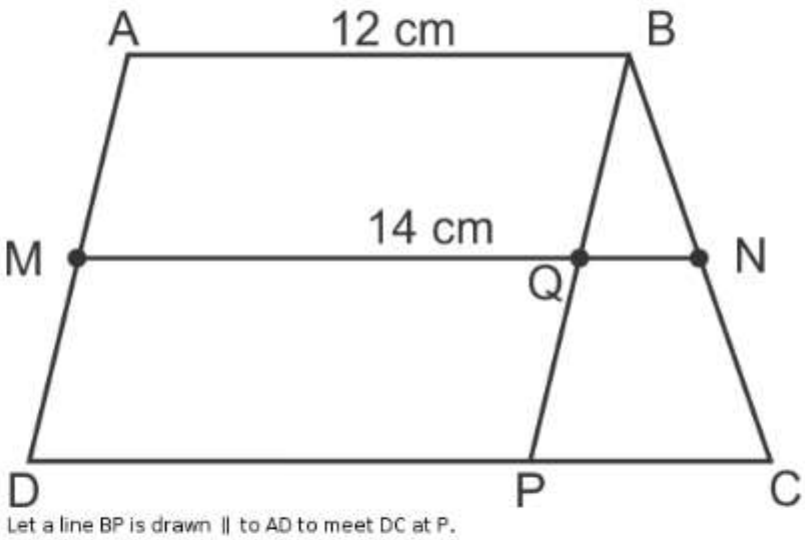


In a parallelogram ABCD,
$\angle OAB = \angle OCD$
In $\triangle OCD$
$\angle OCD + \angle COD + \angle ODC = 180°$
$\angle COD = 90°$
$\angle ODC = 50°$ (given)
$\angle OCD = 180° - 90° - 50° = 40°$
$\Rightarrow \angle OAB = \angle OCD = 40°$
Hence, correct option is (a).

Q9

ABCD is a trapezium in which AB ∥ DC. M and N are the mid – points of AD and BC respectively.
If AB = 12 cm, MN = 14 cm, then CD =
(a) 10 cm
(b) 12 cm
(c) 14 cm
(d) 16 cm

Solution



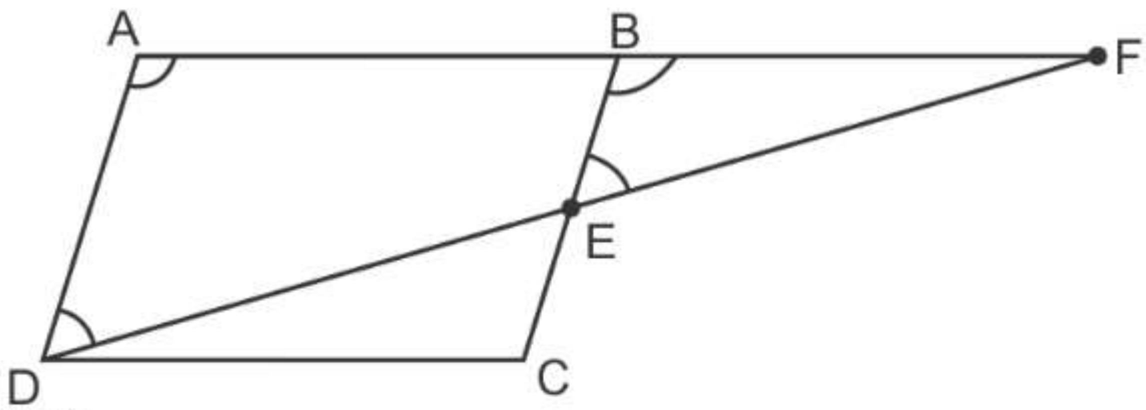
Let a line BP is drawn $\parallel$ to AD to meet DC at P.
ABPD is a parallelogram.
AB $\parallel$ PD, AD $\parallel$ BP
So AB = DP
Let BP cuts MN at Q.
MQ is also $\parallel$ to AB $\parallel$ PD
So AB = MQ = PD = 12 cm ....(1)
QN = MN − MQ = 14 − 12 = 2 cm
Consider $\triangle$BPC.
Q and N are the mid − points of BP & BC, and the line joining them QN $\parallel$ PC.
Then by property, $\frac{QN}{PC} = \frac{1}{2}$
$\Rightarrow$ PC = 2QN = 2 × 2 = 4 cm
Now, DC = DP + PC
DP = 12 cm [From (1)]
$\Rightarrow$ DC = 12 + 4 = 16 cm
Hence, correct option is (d).

Q10

ABCD is a parallelogram, and E is the mid − point of BC.
DC and AB when produced meet at F. Then, AF =

(a) $\frac{3}{2}$AB
(b) 2 AB
(c) 3 AB
(d) $\frac{5}{4}$AB

Solution



BC || AD

$\Rightarrow$ BE || AD

Now, consider $\triangle FAD$

BE || AD

Also $\frac{BE}{AD} = \frac{1}{2}$

In $\triangle FBE$ and $\triangle FAD$,

$\angle FAD = \angle FBE$ {Corresponding angles}

$\angle ADF = \angle BEF$ {Corresponding angles}

$\angle F = \angle F$ {common}

Hence, $\triangle FBE \sim \triangle FAD$

$\Rightarrow \frac{BF}{AF} = \frac{BE}{AD} = \frac{1}{2}$

$\Rightarrow 1 - \frac{BF}{AF} = 1 - \frac{1}{2}$

$\Rightarrow \frac{AF - BF}{AF} = \frac{1}{2}$

$\Rightarrow \frac{AB}{AF} = \frac{1}{2}$

$\Rightarrow AF = 2AB$

Hence, correct option is (b).

Q11

Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 45°$, then $\angle B =$

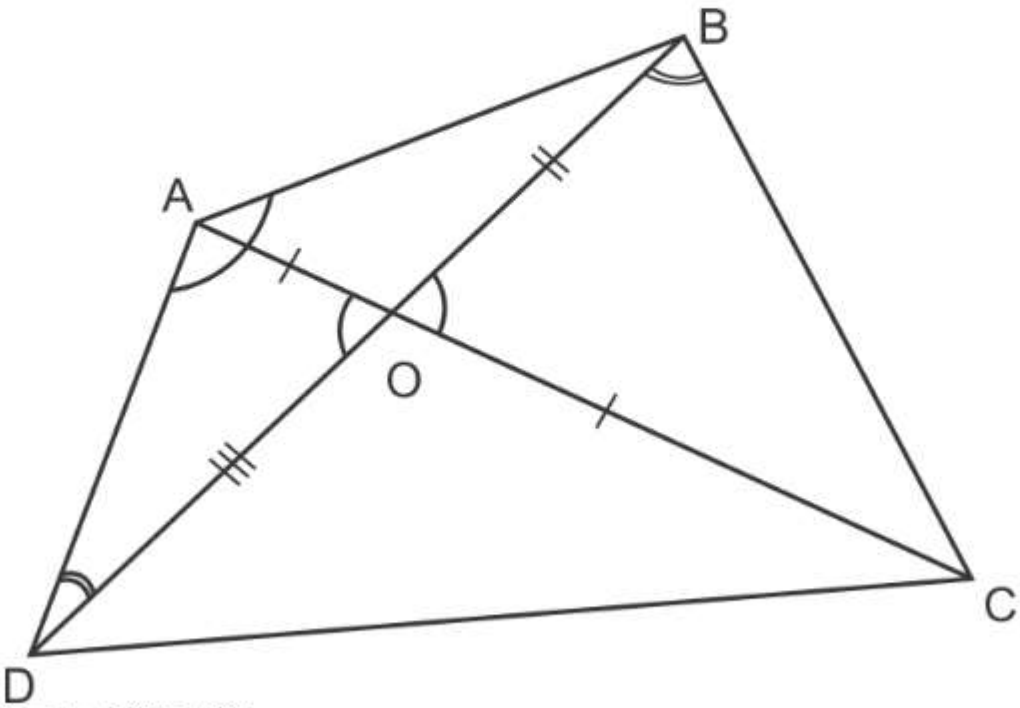
(a) 115°

(b) 120°

(c) 125°

(d) 135°

Solution

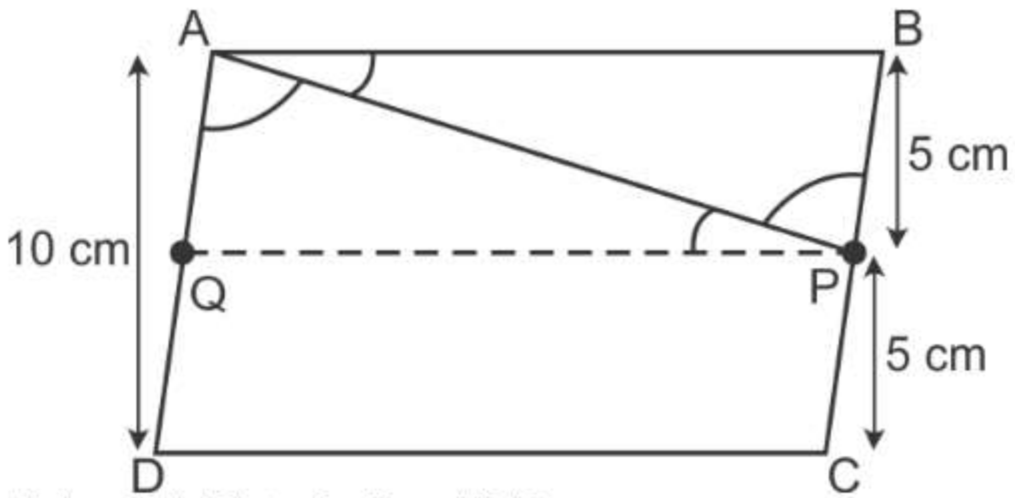


Consider $\triangle AOD$ & $\triangle COB$,
$AO = CO$ {Diagonals Bisects each other}
$OD = OB$ {Diagonals Bisects each other}
$\angle AOD = \angle COB$ (opposite angles)
So, by SAS property, $\triangle AOD \cong \triangle COB$
$\Rightarrow \angle ADO = \angle CBO$ ...(1)
$\angle ABD = 180° - \angle A - \angle ADO$ (in $\triangle ADB$)
$= 180 - 45° - \angle ADO$
$\angle ABD = 135° - \angle ADO$ ...(2)
$\angle B = \angle ABD + \angle CBO$
Putting values From eq (1) and (2)
$\angle B = 135° - \cancel{\angle ADO} + \cancel{\angle ADO}$
$\angle B = 135°$
Hence, correct option is (d).

Q12

P is the mid – point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. If AD = 10 cm, then CD =
(a) 5 cm
(b) 6 cm
(c) 8 cm
(d) 10 cm

Solution

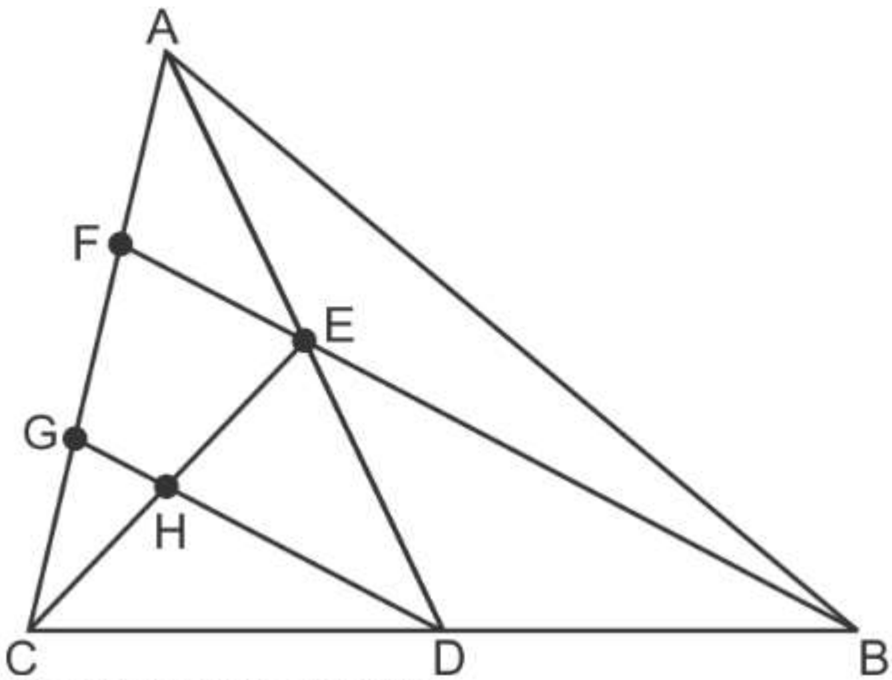


Let a line parallel to AB is drawn from P to meet AD at Q.
PQ || AB || DC
Q is also mid – point of AD.
Now, consider parallelogram ABPQ.
$\angle PAQ = \angle APB$ (Alternate angles)
Also $\angle PAQ = \angle BAP$ (Given)
$\Rightarrow \angle APB = \angle BAP$
So $\triangle ABP$ is isosceles $\triangle$.
$\Rightarrow BP = AB$
i.e. $AB = \frac{10}{2} = 5$ cm
Hence, correct option is (a).

Q13

In ΔABC, E is the mid-point of median AD such that BE produced meets AC at F. If AC = 10.5 cm, then AF =
(a) 3 cm
(b) 3.5 cm
(c) 2.5 cm
(d) 5 cm

Solution



A line DG is drawn parallel to FE to meet AC.

FE ∥ DG and FE ∥ GH

Now, consider $\triangle ADG$.

E is the mid – point of AD and EF is line From E ∥ to Base DG.

So by Property, it will meet AG at its midpoint

i.e. F is midpoint of AG.

$\Rightarrow AF = FG$ .....(1)

Now, consider $\triangle FBC$ & $\triangle GDC$

FE ∥ GH and FE ∥ GD

D is mid – point of BC.

$\Rightarrow \frac{DC}{BC} = \frac{1}{2}$ ....(2)

Because $\triangle FBC \sim \triangle GDC$,

$\Rightarrow \frac{GC}{FC} = \frac{1}{2} \Rightarrow FC = 2GC$

OR $FG = GC$ .....(3)

From equations (1) and (3),

$AF = FG = GC$

$\Rightarrow AF = \frac{AC}{3} = \frac{10.5}{3} = 3.5$ cm

Hence, correct option is (b).

Exercise 13.73

Q1

In a quadrilateral ABCD, $\angle A + \angle C$ is 2 times $\angle B + \angle D$. If $\angle A = 140°$ and $\angle D = 60°$, then $\angle B =$

(a) 60°

(b) 80°

(c) 120°

(d) None of these

Solution

In a quadrilateral ABCD,

$\angle A + \angle B + \angle C + \angle D = 360°$ ...(1)

Now, $\angle A + \angle C = 2(\angle B + \angle D)$ (given) ....(2)

Also, $\angle A = 140°$ $\angle D = 60°$

Putting value of $(\angle A + \angle C)$ from eq. (2) in eq. (1)

$2(\angle B + \angle D) + \angle B + \angle D = 360°$

$3(\angle B + \angle D) = 360°$

$\Rightarrow \angle B + \angle D = 120°$

$\Rightarrow \angle B + 60° = 1200°$

$\Rightarrow \angle B = 60°$

Hence, correct option is (a).

Q2

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle ABD = 50°$, then $\angle DPC =$

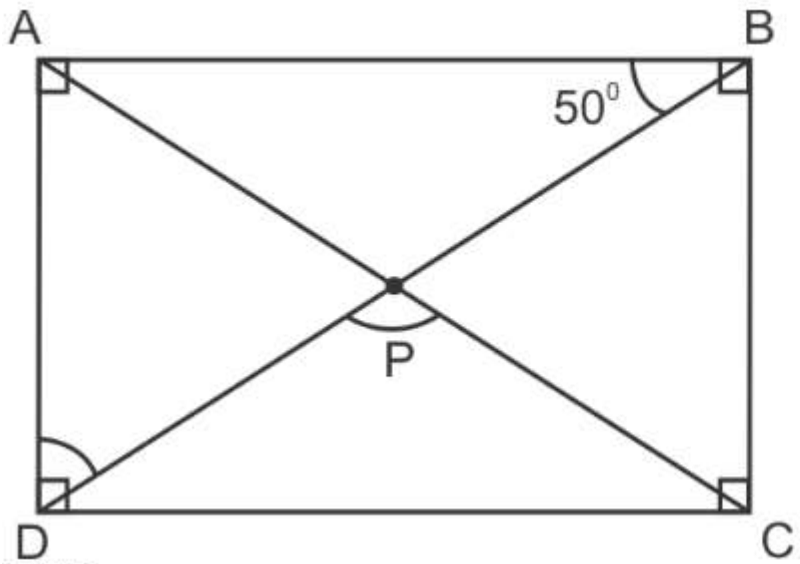
(a) 70°

(b) 90°

(c) 80°

(d) 100°

Solution



In $\triangle ABD$,

$\angle BDA + \angle ABD + \angle DAB = 180°$

$\angle ABD = 50°$ and $\angle DAB = 90°$

$\Rightarrow \angle BDA = 180° - 90° - 50° = 40°$

Consider $\triangle ABD$ & $\triangle BAC$

$AD = BC$, $\angle DAB = \angle ABC = 90°$, $BD = AC$

Hence, by RHS property $\triangle ABD \cong \triangle BAC$

$\Rightarrow \angle ABD = \angle BAC = 50°$

Now, consider $\triangle ABP$

$\angle PAB + \angle PBA + \angle APB = 180°$

$\angle PAB = \angle BAC = 50°$

$\angle PBA = \angle ABD = 50°$

$\Rightarrow \angle APB = 180° - 50° - 50° = 80°$

Now, $\angle APB = \angle DPC$ (Opposite angles)

$\Rightarrow \angle DPC = 80°$

Hence, correct option is (c).